Communication In Organizations: Olgarchies, Hierarchies And Committees

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This paper investigates the performance of three forms si organizations: hierarchies, oiligarchies and committees. Sah and Stiglitz il3S6) show that hierarchies and polyarchies differ in their information $\underset{\sim}{\text { a }}$ assiang abilities in terms of type-I and type-II errors. with "falliria" agents makirg decisions. This paper modifies the structure of a polvircliv and cails it ar oligarchy and examines this notion in terms of an inonfilete ineormation game where players receive private signals about lig si.ti ni naiure. r hierarihy is defined in terms of authority and the aminior $2:$ :ominunicatien allowed and it is shown that the statistical errors vary rieroding on the particular design of the organization. We show that hierir.inisis mav iead tc bet:er information processing in terms of minimizing outh types n: statistical errors but polyarchies have an advantage in terms of time required to reach a decision. We, also, contrast hierarrhical decision making with that of committees. This leads us to suggest that the information requirements for a "good" hierarchy are stringent and if such? information is not available, committees present a suitable alternative. we also disciuss the performance of these organizations when individuals ao no: share the same preferences and show that the position of individuaiz georme important.
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1. Introduction

In a series of papers Sah and Stiglitz(1985.1986.198S) have investigated communication witrin organizations. Their approach has been that human beings by their very nature are fallible in their decision making and the question of the structure of decision making within organizations must be concerned with minimizing these errors. In Sah and Stiglitz(1986) they study two systems hierarchies and polyarchies and examine the difference in the quality of decision making. Polyarchies constitute a system where authority to take decisions is delegated to individuais within an organization. As such, they enjoy a fair degree of autonomy. In contrast a hierarchy exhibits more centralized decision making with only a few individuals at the top of tne bureaucracy authorized to make decisions. while others at lower levels provide support to those in authority.

In their model individuals evaluate projects with a net benefit, which can oe positive or negative. The choices they have is to accept or reject. projects and they do so with a given probability. Obviously, the best decision would be to accept all projects with a positive net benefit and reject all other projects. However, individuals are not perfect in terms of their ability to evaluate projects and to communicate with each other and this is reflected in their probabilistic acceptance of projects. Of course, their decision to accept projects is not entirely random and it is assumed that they are more likely to accept projects of better quality, i.e., higher net benefits. In a polyarchy individuals take decisions to accept or reject according to the given probability. In a hierarchy, in contrast, a project if accepted is passed on to a higher level where it is again evaluated and then accepted or rejected. Consequently, the probability of a project being
accep:ea in a hierarchy is lower than the probability of being accepted in a poiyarchy This shows up in Proposition 1 in Sah and Stiglitz(1986) where they show that a polyarchy accepts a larger proportion of projects than a hierarchy and that hierarchies and poiyarcnies differ in terms of Type-i and Type-i i errors Consequently no conciusions can be drawn about which system is better. Such conclusions depena upon the preferences of the designer of the organization.

One drawback of this model is that individuals take no note of the decisions taken by their counterparts in the lower rungs of the organization. Thus, in a two person hierarchy the individual on top does not take into account that if a project reaches him it has already been accepted at the lower level and proceeds with his evaluation ignoring the implications of this fact. Koh(1992) makes an attempt to remedy this by considering a more general model In his model individuals receive private signals, on a continium, about the quality of projects and the problem for an individual is to determine the cut-off point beyond which projects are to be accepted. This is in line with Sah and Stiglitz(1986) as this will give rise to a probability of acceptance: knowledge of the distribution of the signal would allow us to ascertain the probability that an individual would accept a particular project. Koh shows that in a two person hierarchy the second individual will optimally choose a lower cut-off point. If a project does reach the second tier of a hierarchy then this person, aware that the project , has been evaluated and accepted at the lower level, applies a less stringent criteria for acceptance.

Koh's approach, where he considers the root of fallibility to lie within the nature of information available to a single individual, is a notion we find
appealing What is less appealing is the nature of his analysis. In both Sah and Stiglitz and Koh individuals are only allowed to send messages once The problem then becomes one of finding the optimal cut-off points in a rierarchy and a polyarchy from the folnt of the central planner or the designer of the organization in Koh

Bull and Ordover (1987) use the Sah and Stiglitz approach to investigate the link between the structure of organizations and that of markets. They view the Sah and Stiglitz approach as being complementary to the standard incentives approach to the internal organization of the firm. As an example of the standard approach they cite Lambert (1986). We are sympathetic to the view that, apart from the incentives of the members of the organization to undertake actions in the interests of the owners, the architecture of the organization is an important element in the welfare of the owners of the firm. If communication were unlimited and costless the structure of the firm would be unimportant since any action could be implemented by the revelation principle in any structure. However, as Melumad, Mookherjee and Reichelstein (1996) show, in the case of limited communication the structure of the organization is important in achieving the objectives of the principal

The point, though, is that an analysis devoid of any consideration of the incentives of the members of the organization in reaching various decisions can be misleading. Thus, we will include costs involved in reaching various decisions in our analysis and we also focus on the strategic interactions between the members of the organization. Our view is that a hierarchy affords better and sustained communication than a polyarchy. Whether such communication is, indeed, undertaken depends on the predisposition of the individuals within a hierarchy. A polyarchy on the other hand symbolizes
more individualistic decision making inus the proper approach, in our view. of studying decision making in a hierarcny is to consider the strategies and options available in a hierarchy for communication and to investigate to what extent these possibilities are exploited by individuals to achieve better decisions.

Another method of reaching decisions is through committees. Sah and Stiglitz (1988) provide a model of committees which is similar to their model of hierarchies. It consists of a group of individuals who vote on a project according to a pre-assigned probability Given a majority rule one can then calculate the probability that any particular project will be accepted. This model, of course, shares the same problem with that of hierarchies in that there is no scope for communication. We examine the possibility of decision making through committees through our model. Since there are only two persons in our model we cannot investigate voting as a means for reaching decisions. We feel that an integral part of decision making through committees is requirement of a large measure of consensus in decisions reached. This we ensure by requiring unanimity on decisions reached. Our model allows a greater degree of communication than hierarchical decision making and, as such, captures the essence of decision making through committees:

A further consideration in the design of structures within an organization would be that of authority. The word authority has been used in a number of different ways (Beckman(1988), Katzner(1992)). It could imply a supervisory relationship with individuals in the higher ranks seen as enjoying more authority. It could also be used in relation to the actions a person is allowed to carry out. -Thus, an individual may be allowed to accept a project
without reference to a superior but roi allowed to reject a project in that case we will say that the particular individual has the authority to accept. The rank of individuals within an organization will be discussed in terms of a supervisory relationship. In a hierarchy the individuals in a higher rank will be said to supervise those in lower ranks. Our use of the word supervise does not conform to common usage and it only describes an individual with a larger array of actions, and thus more authority it is more difficult to talk about authority in committees but we could use it to designate the individual who starts off proceedings. This stems from the discovery that the individual who moves first can, under some situations, implement decisions to his liking.

The discussion so far suggests that an investigation into the decision making properties of hierarchies and polyarchies should be conducted in the context of an incomplete information game and that is what we propose to do in the rest of the paper. We modify the structure of polyarchies in Sah and Stiglitz and call it an oligarchy(Wu 1989). The motivation for this modification is discussed in the next section where we present our models of the three different organizational structures. In section 3 we compare hierarchies and oligarchies. We show that, in terms of statistical errors, a hierarchy with limited communication capabilities and an authority structure where the supervisee is only allowed to reject projects will perform as well as an oligarchy. If more communication is allowed in this hierarchy the results can be better or worse, depending on the degree of impatience of the players. If the authority structure is changed then we show that, even with limited communication, a hierarchical structure out-performs an oligarchy. If individuals within an organization do not share the same goals then it becomes more difficult to design a hierarchy to suit one's objectives. The
position of individuals within a nierarchy becomes importart.

Section 4 is devoted to $\exists$ comparison of committees witn hierarchies and oligarchies. We show that all outcomes which can be achieved with a hierarchy or an oligarchy can be achieved with a committee However. these nutcomes could be achieved with less communication and delay in hierarahies and oligarchies. We show that the designing of a good nierarchy requires knowledge of the evaluation skills of the individuals within the organization. If such information is not available. committees present a suitable alternative if the indiriduals are patient. Committees would always reach the optimal decision, in terms of minimizing statistical errors. if the costs of delay are sufficiently low If individuals are biased then the situation is the same as that in a hierarchy. There is a first-mover advantage and we have to take that into account. In such a situation oligarchies could become attractive Section 5 provides the conclusion.

## 2. Model

There are two individuals 1 and 2 who can be used to reach a decision, accept (A) or reject (R), on a project which can either be successful, $S$, or unsuccessful, $U$. The payoffs from this project will be expressed in terms of losses from Type-I and Type-II errors. Thus the loss from accepting an unsuccessful project is a while the loss from rejecting a successful project is $c^{2}$. These costs reflect the effect of compensation received from the employer for the services rendered. It is possible that accepting an unsuccessful project may lead to a lowering of the firm's profits and this could lead to lower payoffs for the employee. This could, also, happen if the employee's salary is linked with the proportion of failures of projects
which the employee had designated for acceptance. If this nethod of compensation is used the same would be true for rejecting successful projects.

Further, each individual receives a private signal $s \in\{g(g o o d i . b(b a d)\}$ about the viability of the project and the probability of receiving the signal $g$ is $\pi$. Each individual has a common prior on the probability that the project is successful which is given by $\zeta$ and given the joint distribution between the states of nature and the private signai can compute the posterior probabilities $\zeta(S \mid g)$ and $\zeta(S \mid b)$ which will be called $\pi_{g}$ and $\pi_{b}$. We can interpret the posteriors as the evaluation skilis of the individuals. An individual with a high $\pi_{g}$ and a low $\pi_{b}$ would be comparatively good at distinguishing between projects. Similarly. if the signals received by the two individuals were to be known the resulting posteriors would be $\pi_{g g}, \pi_{g b}$ and $\pi_{b b}$.

We will be studying three different methods of reaching decisions committees, hierarchies and oligarchies within this framework. The hallmark of a committee is that it requires broad degree of consensus to reach decisions. This we model by a game where individuals $i$ and 2 have to reach a decision on the particular project by sending a series of messages to each other. Thus individual 1 may start the game by sending a message $m_{1} \in\{A, R\}$ to which individual 2 must respond with a similar message. If 2 agrees with l's message, both sending the message $A$ for example, the game ends with a decision to accept the project, otherwise they both incur a cost of delay, by a factor $D(D>1)$, and the game continues till they can agree on a decision. A period or a round of communication is defined by the time required for a individual to propose a decision and for another to react to


Figure 1 (a) Committee with two rounds of communication (G10 and G11). (b) A hierarchy with two rounds of communication(G2).

A possible variation on this structure is to allow the committee a certain number of periods to reach a decision we will model this by saying that if at the end of $k$ periods the two individuals cannot agree on a decision then the game ends without any decision being taken. The structure of the game, for decision to be reached within two periods, is shown above (Figure 1(a)) The purpose of restricting the game to two or any other number of periods is due to the desirability of reaching a quick decision. This is ill served if individuals do not reach a decision at all. Since individuals bear some cost of error from either of the decisions $A$ or $R$, not reaching a decision would be particularly attractive if it were not penalized. Therefore we will assume that there is a cost $k$, which applies to both individuals, if a decision is not reached at the end of the allotted time. Since the organization has been put in place to reach decisions on the acceptance or
rejection of projects we can be safe in presuming that $k$ would be higher than $a$ or $b$.

The d.stinguishing feature of a hierarchy is that there are people in supervisory positions and these people are required to act only if certain actions have taken place at the lower levels. Within our framework we could design a hierarchy where individual $l$ chooses between accepting and rejecting. If the project is accepted it is then passed on to individual 2 who then decides on whether to accept or to reject. Thus there are two decisions to be made when designing a hierarchy: how many rounds of communication to allow and based on what recommendation should the decision reach the higher level. An example is shown in Figure $1(b)$.

Here, two rounds of communication are allowed. Individual 1 can end the game by rejeciing the project. If he accepts then individual 2 can either countermand his decision by rejecting and allowing 1 a further round of communication, or he can agree with 1 and end the game. If $l$ still insists on accepting then 2 ends the game by accepting or rejecting. We will assume that costs of delay are incurred each time there is a aisagreement and further communication is necessary. The exception will be at the last stage where 2 , the person in charge, takes the final decision. It would be possible, one possibility among many, to design a hierarchy with two rounds of communication with 1 ending the game by accepting the project. In our terminology individual 2 supervises 1 who only has the authority to reject.

We could interpret 1 and 2 as two divisions within a firm, production and accounts. Consider a situation where the organization is trying to decide whether to buy a new piece of machinery. It is quite plausible that the $i$,
the production division is allowed to reject this considerdtiun without referral to accounts. If, however, it decides to buy this equipment it would rave to submit its proposal to 2 , accounts. 2 may accept or reject this proposal. If it is rejected 1 could request a reconsideration and then 2 would have the final verdict. Here accounts would be seen to have more authority than production.
in the next section we will compare the quality of decision making in hierarchies and oligarchies. We shall do so by, first, considering a series of examples of different types $c^{r} h$ erarchies. Before we do that let us look at decision making within an oligarchy. A solitary individual will have to hase hir toci=ion or the signal he receivec. As a Bayesian his decision will be
$A$ if $\zeta\left(S \mid s ;>\frac{a}{a+c}, R\right.$ if $\zeta(S \mid s)<\frac{a}{a+c}$,
$A$ or $R$ if $\zeta(S \mid s)=\frac{a}{a+c}$ for $s \in\{g, b\}$.

This decision rule reflects the information available to a solitary individual ${ }^{3}$. The first term shows that the project would be accepted if the value of the posterior, given a signal $g$, is higher than the cost of accepting a successful project as a proportion to total costs. Sah and Stiglitz and Koh envisage a world where if a project is rejected it is available to others for acceptance. Thus in a polyarchy if a project does become available it is possible that it has been rejected by someone else and the decision maker in a polyarchy should take that into account when reaching a decision. However this raises the possibility of strategic acceptance of projects which is similar to the literature on patent races in the field of research and development. Since we wish to avoid such
complications we will assume that there is a single project available and if a project is rejected it is no longer available for others to accept. Since our aim is to study individual decision making versus bureaucratic decision making this approach serves us wcll.

As an example consider employees in a bank who are charged with the responsibility of approving loan applications. The usual approach seems to be one where an individuai reaches a judgment about the quality of a loan application. If the particular employee considers the quality to be good he has to refer the application to his superior for final approval. We would suggest that an applicant who has been turned down cannot turn around and try his luck with some other employee. Of course, he could certainly try his luck with another bank. ${ }^{4}$ If a large number of employees are available then a further question to be asked would be the number of employees an individual supervisor should have below him. ${ }^{5}$ This decision could involve the time required to do an evaluation. In our model it is communication which takes time not evaluation. ${ }^{6}$

Finally, it could be supposed that our model is quite special. First, we consider a situation where the individuals receive two signals and can send only two messages. We could increase the number of signals and messages. The analysis would become more complicated but the results would not change significantly. The crucial assumption is that the individuals' message space is of a smaller dimension than the signal space. As long as that is the case communication will take time and there will be a difference in the performance of the different organizations. We also consider only two individuals. Our results on hierarchies and oligarchies could be extended to more than two individuals. The problem would be with committees. With more

Fhar inv andiviquals there housi de an eiement of cheap talk and the analysis would become difficult. The last concern is with the nature of decisions reached In a large number of situations, like a firm deciding on its cutput, we art concerned with more than two decisions. However, our mude, ovila be used to reach decisions. For example, a firm deciding on its o.itani zuld decide to produce the largest output that is accepted

The notion of equilibrium we will use will be a perfect Bayesian equilibrium and it will be a sequential equilibrium by virtue of there being only two players who could be of two types The proofs of the first two theorems are shown in the appendix. The rest of the proofs are similar and not shown.
3. Hierarchies versus Oligarchies

We will begin by looking at a hierarchy (Figure 2, G1) with one round of communication. Individual $I$ goes first and says $A$ or $R$. If he says $R$ the project is rejected and if he says $A$ it is passed on to the higher level where a decision is made to accept or reject the project. Note that since the game ends at the end of the first period there are no costs of delay. -


Figure 2.G1 and G3
Figure 3.

We will assume that
$\pi_{g g}>\pi_{g b}>\frac{a}{a+c}>\pi_{b b}$

Thus, if each inaividual's private signal were to be observed, the optimal decision would be to accept the project in the events $(g, g)$ and ( $g, b$ ) and to reject in case of the event $(b, b)$. The situation is shown in Figure 3 we will later comment on the equilibria if condition (2) were changed to
$\pi_{g g}>\frac{a}{a+c}>\pi_{g b}>\pi_{b b}$.

As is the standard approach in dealing with a game of incomplete information we will convert it into a game of imperfect information by saying that there can be two types of $l, l(g)$ or $l(b)$. Similarly, there will be two types of 2. $2(g)$ and $2(b)$. The strategies of 1 is to send the message $A$ or $R$ depending on the signal he receives. 2's strategies are the same except that 2 has to exercise his options only if 1 accepts. The payofis for 1 and 2 are in terms of Type-I and Type-II errors. If the decision is A the payoff for i is $a\left(1-\quad \zeta\left(S \mid s_{1}, m_{j}\right)\right)$ where $s_{i}, m_{j} \in\{A, R\} ; \zeta\left(S \mid s_{i}, m_{j}\right)$ is the probability that the project is successful if the individual $i$ receives a signal $s_{i}$ and observes a message $m_{j}$ and so $a\left(1-\zeta\left(S \mid s_{i}, m_{j}\right)\right)$ is the expected loss from accepting an unsuccessful project. The loss from rejection is ç(S|si, $\left.m_{j}\right)$ Of course if 1 rejects a project he does not get to observe 2 's reaction and the payoff would change appropriately. The possible equilibria for this game are shown in the following theorem.

Theorem 3.1: There are three possible equilibria
(i) $1(g): A, 1(b): A, 2(g): A, 2(b): A$ for $\pi \geq \tilde{\pi}$,
(ii) $1(g): A, 1(b): R, 2(g): A, 2(b): A$ for $\pi \leq \tilde{\pi}$,
(iii) $1(g): A, 1(b): A, 2(g): A, 2(b): R$ for $\pi \leq \tilde{\pi}$, where
$\tilde{\pi}=\frac{\frac{a}{a+b}-\pi_{b b}}{\pi_{g b}-\pi_{b b}}$

The first. equilibrium shows a situation where 1 always accepts the project and so does 2. For $1(g)$ is a dominant strategy since there are only two possible events which could occur $(g, g)$ and $(g, b)$ and both of which merit acceptance of the project. In $l(b)$ 's case the two possible events are $(b, g)$ and $(b, b)$. The project should be accepted in the first instance and rejected in the second. Given $z^{\prime} s$ equilibrium strategy of always accepting, $l(b)$ has a choice between accepting or rejecting the project which he does by weighing the possible errors and the associated probabilities. From figure 13 we can interpret $\tilde{\pi}$ as the ratio of the cost of accepting if both players think that the project is bad to total costs of statistical errors. The denominator can be written as the sum of the cost of rejecting if the event is $(g, b), \pi_{g b}-\frac{a}{a+b}$, and the cost of accepting if the event is $(b, b)$. $\frac{a}{a+b}-\pi_{b b}$. Thus, if $\pi \geq \tilde{\pi}, A$ is the optimal decision. $2(g)$ should always accept for reasons similar to $1(g)$. $2(b)$ cannot gather any new information from l's message since both types of 1 send the message $A$ and, therefore, decides to accept the project for the same reason as $l(b)$.

In equilibrium (ii) the situation is reversed for $l(b)$. Now, as before, 2 accepts whenever 1 does but the probability of the event $(b, b)$ is high enough to warrant rejection of the project. Thus, $I(g)$ and $I(b)$ separate. Given this strategy, 2 of both types know the identity of 1 when it is their turn to decide and consequently always accept. In (iii) $2(b)$ rejects given the low probability of the event $(g, b)$. Given this equilibrium strategy 1 of
botn types decide to accept. It should also be noted that we do not need to specify out of equilibrium beliefs for this game because such beliefs would oniy be required if both individuals were, in equilibrium, expected to send a particular message but did not. There are two ways this could happen First, if both types of player 1 were supposed to say $A$ but $R$ was observed instead However, in such a situation the game would have already ended and beliefs would have no effect on strategies. The other way is if both players were supposed to say $R$ but said $A$. This is not possible since player 2 type $g$ would always say $A$ and therefore so would $l(g)$. So both types of player 1 saying $R$ is not possible no matter what the beliefs. In the games studied later, out of equilibrium beliefs will matter and there we will assume that. if in equilibrium any individual were required to send the message $A$ but $R$ were observed, then the deviating individual is type $b$ and the other way around. These beliefs satisfy the Cho and Kreps (1987) intuitive criterion. ${ }^{7}$
it might be presumed, that given that there are a number of possible equilibria, the outcomes of decisions in a hierarchy would be different from that in an oligarchy in terms of Type-I and Type-II errors. The following proposition shows this presumption to be wrong.

Proposition 3.1: The Type-I and Type-II errors for a 1 round hierarchy with 1 having the authority to reject are equal to that in an oligarchy.

Proof: There are two possible cases (i) $\pi_{g}>\pi_{b}>\frac{a}{a+c}$ (ii) $\pi_{g}>\frac{a}{a+c}>$ $\pi_{\mathrm{b}}$. In (i) an oligarchy will always accept the project. If individual 1 (or 2) receives the signal $g$ he would always accept for both cases. If he receives $b$ he knows that the other individual may have received $g$ or but his own signal, being independent of the other signal, gives him no information. Thus $\pi_{b}=\pi \pi_{g b}+(1-\pi) \pi_{b b}$, so that the condition
$\pi_{b}>\frac{a}{a+c}$
$\Rightarrow \pi \pi_{g b}+(1-\pi) \pi_{b b}>\frac{a}{a+c}$
$\Rightarrow \pi>\frac{\frac{a}{a+c}-\pi_{b b}}{\pi_{g b}-\pi_{b b}}$

This, however is the condition for equilibrium (i) in which case the project is always accepted. For (ii) $\pi_{b}<\frac{a}{a+c}$ which is equivalent to $\pi<\tilde{\pi}$. There are two possible equilibria which have the same outcomes. Thus, the outcomes in a hierarchy, depending on the value of $\pi$, are exactly the same in an oligarchy Consequently, the type-I and type-II errors are the same.

The reason behind this result is that in a one round hierarchy there is limited scope for communication. Even though 1 has two signals he is limited by the nature of the game to be effectively left with just one signal. A. The problem is that he has to signal what information he has received with his recommendation $A$ or $R$. The only way he can provide an informative signal is by using a different recommendation for the two signals he receives. One way of doing this would be to say $A$ if he observes $g$ and $R$ if he observes $b$. However, if he says $R$ the game ends: even though 2 might have received the signal $g$ and the right decision would be $A$, there is no way to undo the damage. So in any equilibrium either 1 pools and his message is uninformative or he separates but 2 cannot use this information. Thus a hierarchical structure of organization is by itself not a guarantee of better quality decision making and careful consideration has to be given to the exact structure of the hierarchy. This result is also at odds with Sah an Stiglitz's result that the errors are different for hierarchies and polyarchies and is a consequence of considering the costs of reaching decisions. We would, however, have to be careful about making comparisons
between our model and that of San and Stiglitz since trey are not similar

Our earlier investigations have shown that a hierariny with limited communication is no better than an oligarchiy in iexis of minimizing statistical errors. We might presume that a hierarchy wnicn allows more communication would do better. We shall now go on to consider a game which has a similar structure with our previous game with the added alstinction in that two rounds of communication are allowed. The strategy space for 1 and 2 will now be $\{\{A, A\},\{A, R\},\{R, A\},\{R, R\}\}$; the first op:ion refers to the strategy of sending the message $A$ in the first perioa. and in the second period, if 2 responds with $R$ in the first period. The second is the strategy of saying $A$ in the first period and agreeing with 2 if 2 responds with $R$. The third refers to the strategy of saying $R$ in the first period and then $A$ ir the second period. Even though the game would end in the first period if player 1 played this strategy we still need to specify his strategy in the second period to look for sequential equilibria. An enlargement of the strategy space inevitably leads to an enlargement of ine message space and $m_{i}$ now becomes a set rather than a single message. we require that the players use the Bayes rule to update their priors after each message observed. The structure of the game is shown in figure 1. G2

Since the game can now conceivably take more than one period the cost of delay comes into play and an investigation of the various equilibria is broken into two parts where $D \geq \tilde{D}=\frac{c \pi_{g b}}{a\left(1-\pi_{g b}\right)}$ and $D \leq \tilde{D}$. The condition $D \geq \tilde{D}$ implies $\mathrm{Da}\left(1-\pi_{g b}\right) \geq c \pi_{g b}$ which means that it is not worthiwhile to incur the cost of delay to achieve the outcome $A$ if the event $1 s(s, b)$ Thus $\tilde{D}$ serves as a useful benchmark for costs of delay with $D$ higher than $\tilde{D}$ signifying
relatively high costs of delay or impatience on part of the players. Theorem 3.2 shows the equilibria in this case.

Theorem 3.2: For $D \geq \tilde{D}$, there are three possible equilibria. They are
(i) $1(g):\{A, R\}, 1(b):\{A, R\}, 2(g) \cdot\{A, A\}, 2(b):\{A, A\}$ for $\pi \geq \tilde{\pi}$,
(ii) $1(g):\{A, R\}, 1(b):\{R, R\}, 2(g):\{A, A\}, 2(b):\{A, A\}$ for $\pi \leq \tilde{\pi}$,
(iii) $1(g):\{A, R\}, 1(b)\{A, R\}, 2(g) \cdot\{A, A\}, 2(b):\{R, A\}$ for $\pi_{3} \geq \pi \geq$ $\max \left\{\pi_{1}, \pi_{2}\right\}$, where
$\pi_{1}=\frac{\frac{(D-1) c \pi_{g b}}{a+c}}{\frac{(D-1) c \pi_{g b}}{a+c}+\pi_{g g}-\frac{a}{a+c}}, \quad \pi_{2}=\frac{\frac{(D-1) c \pi_{b b}}{a+c}}{\frac{(D-1) c \pi_{b b}}{a+c}+\pi_{g b}-\frac{a}{a+c}}$,
$\pi_{3}=\frac{\frac{a}{a+D c}-\pi_{b b}}{\pi_{g b}-\pi_{b b}}$.

Even though (i) and (ii) in theorem 3.2 look similar to their counterparts in Theorem 3.1, there is an important difference. Here these equilibria depend on the out of equilibrium belief that when 1 says $A$ if 2 counters with $R$ then 1 believes that 2 is type $b$ and therefore says $R$ and ends the game. In (iii) both types of player 1 say $A$ but now $2(b)$ disagrees and then player 1 agrees with 2. The equilibrium outcomes for the possible events are shown in the tabie below. The expressions $\pi_{1}, \pi_{2}$ and $\pi_{3}$ can be interpreted. like before, as the relative costs of sending the two messages. In $\pi_{1} l(g)$ compares the cost of accepting with rejecting. The numerator shows the cost of saying $A$; if 2 is type $b$ he is going to say $R$ and 1 would agree. The cost, therefore, is the error from $R$ for the event ( $g, b$ ) multiplied by the cost of delay. The denominator shows the total cost of both actions. The
term $\pi_{g q}-\frac{d}{a+c}$ is the cost of rejecing when the event is (g.g) $\pi_{2}$, the relevant condition for $1(b)$, can be interpreted in a similar way. $\pi_{3}$ is similar to $\tilde{\pi}$ with the difference being that the cost of wrongly rejecting a project. C. has to be multiplied bv D Plaver 2 does not face imposing delay if he accepts but does so if he disagrees witn $I$

|  | equilibrium outcomes |  |  |
| :--- | :---: | :---: | :---: |
| events | (i) | (ii) | (iii) |
| g.g | A | A | A |
| g.b | A | A | R |
| b.g | A | R | A |
| b.b | A | R | R |

Table 1.

|  | equilibrium outcomes |  |  |  |
| :--- | :--- | :---: | :---: | :---: |
| events | (i) | (ii) | (iii) |  |
| G.g | A | A | A |  |
| g.b | A | A | A |  |
| b. | A | A | A |  |
| b.b | A | R | R |  |

Table 2.

Crom table 1 we can see that equillbrium (i) and (ii) achieve the same equilibrium as a hierarchy with one round of communication Hower there are two other equilibria which do worse. Equilibrium (iii) achieves the same outcomes as (ii) except that if the event is ( $g, b$ ) the outcome $R$ is achieved with delay. In this equilibrium both types of $l$ say $A$ in the first period and so this message has no information content. Thus. we see that if the costs of delay are sufficiently high it is possible thät inferior outcomes are achieved with greater communication. This is stated in proposition 3.2.

Proposition 3.2: The equilibrium outcomes in a two period hierarchy with 1 having the authority to reject can be inferior to that in a one period hierarchy.

Corollary: The equilibrium outcome in a hierarchy can be worse than an oligarchy.

Proof: From proposition 3.1 the outcomes in a one period hierarchy is the same as in an oligarchy. If an outcome is worse than a one period hierarchy
it is worse than an oligarchy

The above result could make us pessimistic about the efficacy of hierarchies in enabling organizations to reach better quality decisions and such pessimism would not be entirely unfounded. Our next two results show that that a stronger case can be made for hierarchies if the cost of delay is sufficiently low

Theorem 3.3 For $D \leq \tilde{D}$ there are three possible equilibria. They are (i) $1(g) .\{A, A\}, 1(b)\{A, R\}, 2(g):\{A, A\}, 2(b) \cdot\{A, A\}$ for $\pi \geq \tilde{\pi}$ (ii) $1(g)\{A, A\}, 1(b):\{A, R\}, 2(g):\{A, A\}, 2(b):\{R, A\}$ for $\pi_{5} \geq \pi \geq$ $\max \left\{\pi_{2}, \pi_{4}\right\}$
(iii) $1(g) .\{A, A\}, 1(b):\{R, R\}, 2(g):\{A, A\}, 2(b):\{A, A\}$ for $\pi \leq \tilde{\pi}$

$$
\text { where } \pi_{4}=\frac{\frac{D^{2} a}{D^{2} a+c}-\pi_{g b}}{\frac{D^{2} a}{D^{2} a+c}-\pi_{g b}+\frac{a+c}{D^{2} a+c} \pi_{g g}-\frac{a}{D^{2} a+c}} \text { and }
$$

$$
\pi_{5}=\frac{\frac{a}{a+D c}-\pi_{b b}}{\frac{\left(D^{2}-1\right) a\left(1-\pi_{g b}\right)}{a+D c}+\frac{a}{a+D c}-\pi_{b b}}
$$

The outcomes are shown in Table 2. From looking at the table we can see that for (i) and (iii) the equilibrium outcomes are the same as in a hierarchy with only one round of communication. Equilibrium (ii) gets the best results but some conditions have to be satisfied. Essentially $\pi$ and $D$ have to be low enough for $2(b)$ to respond with $R$ when $l$ sends the message $A$ but not so high
that if $2(b)$ were to respond with $R, l(b)$ would find it op:imal to end the game in the first period by saying $R$. Thus we have proposition 3 . Also. it might take two periods to reacn a decision in this equilibrium which sits well with the notion that a hierarchy might take longer to reach a decision but would reach better quality decisions than an oligarchy.

Proposition 3.3. If $D \leq \tilde{D}$ there is an equilibrium which achieves the best possible outcome.

Actually, it is possible to do even better with a hierarchical decision structure and this is evident when we study the next game, G3 It has a similar structure to $G l$ except that now the game ends if the project is accepted, otherwise the decision is referred to 2 with 1 s recommendation. The strategy space and payoffs will be similar to that in Gl. The structure of the game is shown in Figure $2(b)$.

As shown in theorem 3.4 this reduces the number of equilibria drastically and in fact manages to achieve the best possible outcome in all possible sases. $l(g)$ would now accept and end the game leaving $I(b)$ to say $R$ to indicate that he has received the signal $b$. Thus 2 is perfectly informed about $l^{\prime}$ s signal when it is his turn to reach a decision and can do so appropriately. Also, this decision is reached within one period so that there are no costs of delay. By appropriately designing the hierarchy it is possible to economize on both statistical costs of errors and costs of delay which is the message of proposition 3.4.

Theorem 3.4: There is only one equilibrium, which is
$1(g):\{A\}, 1(b):\{R\}, 2(g):\{A\}, 2(b) .\{R\}$.

Proposition 3.4: The equilibrium outcomes for this game always achieve the best possible outcome.

This result is the outcome of the different design of the hierarchy. Fron the structure of the posteriors and the cost of statistical errors it shoula be clear that the project should be accepted if one of the two players receive the signal g. Ey giving 1 the authority to accept allows him to use his other recommendation, $R$, to signal his information. The structure 0 : authority should depend on the evaluation skills of the members of the organization. If the posteriors had been different, as in (3), Gl would de the best hierarchical structure.

Till now we have only considered individuals with same preferences wher. dealing with the optimal structure of hierarchies. It is indeed possibıe that the members of an organization do not have the same preferences over outcomes. This could be the result of different compensation schemes or for more fundamental reasons. We will now look at the effect this might have or. the quality of decision making within hierarchies. We will assume that

$$
\begin{equation*}
\pi_{g g}>\frac{a_{2}}{a_{2}+c}>\pi_{g b}>\frac{a_{1}}{a_{1}+c}>\pi_{b b} \tag{3}
\end{equation*}
$$

For the events $(g, g)$ and $(b, b)$ both individuals agree that the outcomes should be $A$ and $R$ respectively. There is conflict of opinion over $(g, b)$. individual 1 believes that the decision should be $A$ while 2 takes the opposite view. This results from 1 and 2 giving a weight $a_{1}$ and $a_{2}$ on wrongly accepting a project with $a_{1}<a_{2}$. The weights on wrongly rejecting $\exists$ project are assumed to be the same for both individuals. We also assume that
these biases are common knowledge

Earlier in our analysis, where both individuals shared the same preferences over outcomes, the optimal decision, in terms of minimizing statistical errors, was never in doubt Now since the two players have different preferences over outcomes. the optimal design of an organization will depend on the preferences of the designer. Let us assume that the preferences of the designer. the manager of the unit, perhads. are parameterized by $a_{0}$ and $c_{0}$. To provide a basis for comparison with our further analysis we will first analyze the problem as to how the manager would maximize his payoff if the two players directly reported their messages to him. Let us designate by $x_{1}, x_{2}, x_{3}$ and $x_{4}$ the probabilities with which the manager would accept the project based on the reports he receives from the two players, i e
$P\left(A \mid m_{1}=g, m_{2}=g\right\}=x_{1}, P\left(A \mid m_{1}=g, m_{2}=b\right\}=x_{2}$
$P\left(A \mid m_{1}=b, m_{2}=g\right\}=x_{3}, P\left(A \mid m_{1}=b, m_{2}=b\right\}=x_{t}$

The payoff for the manager is then

$$
\begin{align*}
& \pi^{2}\left\{a_{0}\left(1-\pi_{g g}\right) x_{1}+c_{0} \pi_{g g}\left(1-x_{1}\right)\right\}+\pi(1-\pi)\left\{a_{0}\left(1-\pi_{g b}\right) x_{2}+c_{o} \pi_{g b}\left(1-x_{2}\right)\right\}+ \\
& \pi(1-\pi)\left\{a_{0}\left(1-\pi_{g b}\right) x_{3}+c_{0} \pi_{g b}\left(1-x_{3}\right)\right\}+(1-\pi)^{2}\left\{a_{0}\left(1-\pi_{b b}\right) x_{4}+c_{0} \pi_{b b}\left(1-x_{4}\right)\right\} \tag{5}
\end{align*}
$$

Simplifying terms we get

$$
\begin{align*}
& \pi^{2} x_{i}\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g q}\right\}+\pi(1-\pi)\left(x_{2}+x_{3}\right)\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g b}\right\}+ \\
& (1-\pi)^{2} x_{4}\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{b b}\right\}+\pi^{2} c_{0} \pi_{g q}+2 \pi(1-\pi) c_{0} \pi_{3 b}+(1-\pi)^{2} c_{0} \pi_{b b} \tag{6}
\end{align*}
$$

By looking at this expression we see that the values of the probabilities of
acceptance that the manager would choose depends on the terms within the curly brackets. Since the manager would minimize (6) he would choose to accept the project with probability one in all circumstances if these terms within the curly brackets turned out to be negative. Simple algebra and a consideration of (3) shows that the terms being negative amounts to $\pi_{b b}>$ $\frac{a_{0}}{a_{0}+c_{0}}$. Since such extreme preferences would not be interesting we will consider two preferences: (i) $\pi_{\mathrm{gD}}>\frac{a_{0}}{a_{0}+c_{0}}>\pi_{b b}$ or (ii) $\pi_{g b}<\frac{a_{0}}{a_{0}+c_{0}}<$ $\pi_{g g}$. Notice that (i) and (ii) will produce the same preferences over outcomes as players 1 and 2 respectively. Thus when we refer to preferences of the manager in future we will refer to $a_{1}$ or $a_{2}$.

The procedure now would be to derive the constraints under which the players would truthfully reveal their signals and then go on to derive the optimum values of the probabilities of acceptance This is done in Appendix B. From (6) the manager with preferences $a_{1}$ would choose $x_{1}=x_{2}=x_{3}=1$ and $x_{4}=0$ if he was fully informed. However as we show this first-best solution cannot be achieved.

Proposition 3.5: The first-best solution for $\pi_{g b}>\frac{a_{0}}{a_{0}+c_{0}}$ is $x_{1}=x_{2}=x_{3}=$ 1 and $x_{4}=0$; and that for $\pi_{q b}<\frac{a_{0}}{a_{0}+c_{0}}$ is $x_{2}=x_{3}=x_{4}=0$ and $x_{1}=1$ Neither of these are achievable through a direct revelation mechanism.

The solution to the direct mechanism problem is given below.
Proposition 3.6: The solution for (i) $\pi_{g b}>\frac{a_{0}}{a_{0}+c_{0}}$ is
(a) $x_{1}=x_{2}=x_{3}=x_{4}=1$ if $\left|\pi\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g b}\right\}\right|<\mid(1-\pi)\left\{a_{0}\right.$
$\left(a_{0}+c_{0} j \pi_{b 0}\right) \mid$,
(b) $x_{1}=1, x_{2}=1, x_{3}=0, x_{4}=C$ if $\left.\mid \pi^{i} a_{0}-\left(a_{0}+c_{0}\right) \pi_{G b}\right\} \mid>$
$\left|(1-\pi)\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{b b}\right\}\right|$ and
$\left|\pi\left\{a_{2}-\left(a_{2}+c\right) \pi_{q G}\right\}\right|<\mid(1-\pi)\left\{a_{2}-\left(a_{2}+c\right) \pi_{q b}\right\} ;$
(c) $x_{1}=1, x_{3}=1, x_{4}=0$ and $x_{2}=1+\frac{(1-\pi)\left\{\exists_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}}{\pi\left\{a_{2}-\left(a_{2}+c\right) \pi_{g q}\right\}}$
if $\left|\pi\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g b}\right\}\right|>\mid(1-\pi)\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{b b} ; i\right.$ and
$\left|\pi\left\{a_{2}-\left(a_{2}+c\right) \pi_{g g}\right\}\right|>\left|(1-\pi)\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}\right|$.
and that for (ii) $\pi_{g b}<\frac{a_{0}}{a_{0}+c_{0}}$ is
(a) $x_{1}=x_{2}=x_{3}=x_{4}=0$ if
$\left|(1-\pi)\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g b}\right\}\right|>\left|\pi\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g g}\right\}\right|$,
(b) $x_{1}=1, x_{2}=0, x_{3}=1, x_{4}=0$ if $\left|(1-\pi)\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{q b}\right\}\right|>$ $\left|\pi\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g q}\right\}\right|$ and $\left|(1-\pi)\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\}\right|<\left|\pi\left\{a_{1}-\left(a_{1}+c\right) \pi_{q b}\right\}\right|$,
(c) $x_{1}=1, x_{3}=0, x_{4}=0$ and $x_{2}=-\frac{\pi\left\{a_{1}-\left(a_{1}+c\right) \pi_{q b}\right\}}{(1-\pi)\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\}}$ if
$\left|(1-\pi)\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g b}\right\}\right|<\left|\pi\left\{a_{0}-\left(a_{o}+c_{0}\right) \pi_{g g}\right\}\right|$ and
$\left|(1-\pi)\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\}\right|<\left|\pi\left\{a_{1}-\left(a_{1}+c\right) \pi_{g b}\right\}\right|$

| events | outcomes |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\left(\pi_{g b}>\frac{a_{0}}{a_{0}+c_{0}}>\pi_{b b}\right) \quad\left(\pi_{g b}<\frac{a_{0}}{a_{0}+c_{0}}<\pi_{g g}\right)$ |  |  |  |  |  |
|  | (a) | (b) | (c) | (a) | (b) | (c) |
| $g . g$ | A | A | A | R | A | A |
| $g, b$ | A | A | $x_{2}$ | R | R | R |
| $b, g$ | A | R | A | R | A | $\times_{3}$ |
| $b, b$ | A | R | R | R | R | R |

Table 3.

The outcomes for the various events are shown below for the purpose of
comparison with different structures of organizations. We should caution that these above results hold for the mechanism we considered. If the principal could choose the costs of the two errors he would make them such that they reflected his preferences and, thus, would be able to reach his preferred outcome in all circumstances This would also be true if the principal had some limited power to impose extra costs on the players in some events.

A curious fact about the analysis so far is that none of the results obtained depend on the position of the individuals within a hierarchy. It makes no difference if we let individual 2 make the first move and allow 1 to respond to $2^{\prime} s$ message. From a cursory glance at organizations this would not seem to be the case. It is felt that individuals at higher ranks within organizations have some special qualifications which make their special positions suitable (Sobel 1992). If individuals are biased in their preferences then their ordering within an organization becomes of importance so that we have to add another consideration to the design of hierarchies beside the rounds of communication allowed and on what decisions the game should be terminated.

Consider the two games, G4 and G5, shown in figure 4 below: (a) is the same as G1 with 1 making the first move and 2 responding while (b) exhibits the opposite situation. The strategies and payoffs remain similar. There are two possible equilibria for each of them which are detailed in Theorem 3.5 and 3.6.
A
A

A
1
ㅁ

A
2
(a)
(b)
Figure 4 G4 and G5

|  | equilibrium outcomes |  |
| :---: | :---: | :---: |
| gvents | $(i)$ | $(1 i)$ |
| $5, g$ | $R$ | $A$ |
| $g . b$ | $R$ | $R$ |
| $D .5$ | $P$ | $A$ |
| $D, b$ | $R$ | $R$ |

Table 4

Theorem 3.5 There are two possible equilibria which are
(i) $1(g)$ A, $1(b): A, 2(g)$ R, $2(b) \quad R$ if $\pi \leq \bar{\pi}$ and
(ii) $1(g)$ A, $1(b): A, 2(g)$ A, $2(b) \quad R$ if $\pi \geq \bar{\pi}$,
where $\bar{\pi}=\frac{\frac{a_{2}}{a_{2}+c}-\pi_{g b}}{\pi_{g g}-\pi_{g b}}$

A comparison with $G 1$ reveals that there is one less equilib-lum Previously there was an equilibrium where the two types of player 1 separated. $l(g)$ said $A$ and $l(b)$ said $R$. This can no longer be an equilibrium because of the biases of the two individuals If this were to be a part of an equilibrium strategy then on receiving the message $A\left(C_{5}\right)$ would accept and $2(b)$ would reject. Given 2 's strategy it is not optimal for $1(b)$ to say $R$; he is better off mimicking $1(g)$ and saying $A$. If he were to adopt this strategy he would get his preferred outcome of $A$ in the event $(\xi, b)$ and $R$ in the event $(b, b)$. So in this game in any equilibria player $l^{\prime}$ 's message is not informative at all Consequently the outcomes are the same as an oligarchy of an individual with bias $a_{2}$. We might presume that things would turn out different if the order of the individuals were to be reversed thus making 2 supervise 1 The equilibria are shown in Theorem 36

Theorem 3.6: There are two possible equillbria winch are
(i) $2(g): R, 2(b): R, 1(g): A, 1(b): A$ if $\pi \leq \bar{\pi}$ and
(ii) $2(g): A, 2(b): R, 1(g) \cdot A, 1(b) \cdot A$ if $\pi \geq \vec{\pi}$,
where $\bar{\pi}=\frac{\frac{a_{2}}{a_{2}+c}-\pi_{g b}}{\pi_{g g}-\pi_{g b}}$

As it turns out the position of the individuals does not make any difference. We can see that the individual in the higher rank always accepts. Thus the choice the individual at the lower rank has is between $A$ and $R$ which again is the same as in an oligarchy This example should make us cautious about relating authority with the position of individuals withan an organization as far as we are concerned with outcomes of such decisions Authority is inextricably linked with the responsibilities and duties of all individuals within an organization In G4 and G5 the outcomes match the preferences of the individual with bias $a_{2}$. The outcomes are shown in Table 4 By comparing with Table 3 we can see that it picks up the first two solutions in (ii). Thus the hierarchical structure under-performs the direct revelation mechanism.

We might expect that the results would be different if the individuals were to play G3 (figure 5, G6 and G7) instead and so they are as shown in theorem 3.7 and 3.8. However, the positioning of the individuals within a hierarchy still do not make any difference though the probability of acceptance is different. Further we saw when we investigated the properties of G3 that it provided the best results which is no longer the case. Whether the designer had preferences of $a_{1}$ or $a_{2}$ he would in some circumstances certain to be disappointed. The outcomes for the games are shown in the table 5 .

Theorem 3.7. There are two possibie equil. : a
(i) $1(\mathrm{~g}): \mathrm{A}, \mathrm{l}(\mathrm{b}): \mathrm{A}, 2(\mathrm{~g}) \mathrm{R}, 2(\mathrm{~b}) \mathrm{R}$ if $\pi \geq \hat{\pi}$ and
(ii) $\mathrm{i}(\mathrm{g}): \mathrm{A}, \mathrm{l}(\mathrm{b}) \cdot \mathrm{R}, 2(\mathrm{~g}) \cdot \mathrm{R}, \mathrm{2}(\mathrm{b}) \mathrm{R} 1 \mathrm{f} \pi \leq \pi$ where
$\hat{\pi}=\frac{\frac{a_{1}}{a_{1}+\pi_{b b}}-\pi_{b b}}{\pi_{g b}-\pi_{b b}}$


|  | equilibrium outcomes |  |
| :--- | :---: | :---: |
| even:s | (i) | $($ ii $)$ |
| $g \cdot g$ | $A$ | $A$ |
| $g . b$ | $A$ | $A$ |
| $b . g$ | $A$ | $R$ |
| $b . b$ | $A$ | $R$ |

Table 5.

Theorem 3.9: There are two possible equilibria
(i) $2(g):$ P. $2(b): R, 1(g): A, 1(b): A$ if $\pi \geq r$ and
(ii) $2(g): R, 2(b): R, 1(g): A, 1(b): R$ if $\pi \leq i$ where
$\hat{\pi}=\frac{\frac{a_{1}}{a_{1}+c}-\pi_{b b}}{\pi_{y,}-\pi_{b b}}$.

Even though neither of the two designs produces perfect results The designer could still prefer one over the other. If for example $a=a_{1}$ then G6(or G7) would be preferred over G4 (or G5). However the outcomes in each case is the same as an oligarchy with the particular tias. We summarize our findings in proposition 3.5.

Proposition 3.7: The position of the individuals do not make any difference in a hierarchy with one round of communization, though the authority of the first individual does. The outcomes are the same as in an oligarchy.

The above proposition is no: universally true as can be seen from the games shown in figure 6 These two games(G8 and G9) are the same as G5 and G6 with an additional round of communcation allowed Theorem 39 shows that for certain specifications of the parameters it is possible to reach the ideal outcomes for $a$ designer with $a=a_{2}$ for $G 8$ However, for $G 9$ the set of equilibria remain the same as those in G6


Figure 6 G8 and G9.

Theorem 3.9 (a) For G8 the equilibria are
(i) $1(g) .\{A, R\}, 1(b),\{A, R\}, 2(g)\{A, A\}, 2(b)\{A, R\}$ for $\pi \geq \max \left\{\hat{\pi}, \pi_{6}, \pi_{7}\right\}$
(ii) $1(g)\{A, R\}, 1(b)\{R, R\}, 2(g)\{A, A\}, 2(b)\{A, R\}$ for $\pi \leq \hat{\pi}, D \geq$ $\frac{a_{2}\left(1-\pi_{g b}\right)}{c \pi_{g b}}$
(iii) $1(g):\{A, R\}, 1(b) \cdot\{A, R\}, 2(g):\{A, A\}, 2(b):\{R, R\}$ for $\pi_{7} \geq \pi \geq$ $\max \left\{\pi_{6}, \pi_{8}, \pi_{9}\right\}$.
(iv) $1(g) \cdot\{A, R\}, 1(b):\{R, R\}, 2(g)\{A, A\}, 2(b):\{R, R\}$ for $\pi_{9} \geq \pi \geq \pi_{8}, D \leq$ $\frac{a_{2}\left(1-\pi_{g b}\right)}{c \pi_{g b}}$.
where $\pi_{6}=\frac{\frac{a_{2}}{a_{2}+D c}-\pi_{g b}}{\pi_{g g}-\pi_{b b}}, \quad \pi_{7}=\frac{\frac{a_{2}}{a_{2}+D c}-\pi_{b b}}{\pi_{g b}-\pi_{b b}}, \quad \pi_{8}=\frac{\frac{(D-1) c \pi_{g b}}{a_{1}+c}}{\frac{(D-1) c \pi_{g b}}{a_{1}+c}+\pi_{g g}-\frac{a_{1}}{a_{1}+c}}$
and
$\pi_{9}=\frac{\frac{(D-1) b \pi_{b b}}{a_{1}+c}}{\frac{(D-1) b \pi_{b b}}{a_{1}+c}+\pi_{g b}-\frac{a_{1}}{a_{1}+c}}$.
(b) For 69 there are two equilibria They are
(i) $2(g):\{A, R\}, 2(b) \cdot\{R, R\}, 1(g):\{A, A\}, 1(b):\{A, A\}$.
(ii) $2(g):\{R, R\}, 2(b):\{R, R\}, 1(g):\{A, A\}, 1(b):\{A, A\}$.

|  | equilibrium outcomes |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| events | (i) | (ii) | (iii) | (iv) |
| $g . g$ | A | A | A | A |
| $g . b$ | A | A | R | R |
| $b . g$ | A | R | A | R |
| $b . b$ | A | R | R | R |

Table 6 .

From Table 6 we can see that a designer with preferences $a_{2}$ would prefer equilibrium (iv) in part (a). However, the existence of this equilibrium is not assured it depends on the value of $\pi$ and $D$. In particular the value of D has to be low. Equilibria (i) and (ii) achieve the same equilibria as oligarchies and so does (iii) but with delay. Thus the message is that designing an optimal hierarchy is much easier if the individuals within an organization share the same preferences. For example Gl will provide the ideal outcomes for a designer with preferences $a_{2}$ without delay. If individuals are biased the designer has to lay stress on the authority and supervisory roles as well as the amount of communication allowed. It might seem that it is possible to improve upon the direct revelation mechanism. The reason behind this is that in the direct revelation mechanism the only control the designer has is over the probability of acceptance. In designing a hierarchy the designer can, through delay, have, an effect on the payoffs
from accepting and rejecting If we allowed the imposition of costs equal to the cost of delay on the two players for the same messages which lead to delay in a hierarchy we would be able to reach the same outcome through the direct revelation mechanism. ${ }^{8}$

Given the difficulties associated with designing an optimal hierarchy when individuals are biased it could be better to use oligarchies instead. The main advantage would be that decisions would reached quickly Also, one would be assured of a diversified portfolio. If the individuals have the same preferences then arranging them as individual decision making units does not impact on the portfolio of projects The two individuals make the same choices However if they are biased then the two individuals would make different choices and if projects were randomly assigned to the two individuals that could mark an improvement over a one round hierarchy

## 4. Committees

Having concluded our discussion on hierarchies it is now time to discuss another form of decision structure namely that of committees. we will investigate in turn the situation where the members of a committee share the same preferences and when they do not. Generally, there are less restrictions in a committee on the messages individuals can send and consequently more equilibria. The only form of committee decision making we will study will be one where two rounds of messages are allowed (figure 1 , G10). It is shown in Gupta (1995) that with unlimited rounds of communication allowed in equilibrium players only utilize two rounds of communication if they share the same preferences; so investigating a game where three rounds of communication are allowed, for example, would be
pointless

There is also no point in investigating a committee with one round of communicasion if there are very high penalties for not reaching an
 second would certainly agree with whatever message the first individual sends As such the decision is left to a single individual. The outcomes of such a decision siructure would then resemble that of an oligarchy. Since our efforts are geared towards finding out how the decision structure facilitates communication within organizations, a one round committee would not be very helpful.

As stated earlier the equilibria for this game will resemble those in Gupta(1995) and here we report the possible equilibria without the proofs and the values of the parameters for which they hold. This is done in Theorem 4.1 and 4.2.

Note that the strategies are different from before. Individual 2 who responds to 1 now has to consider his options for $A$ and $R$. To ease notation we will introduce two new strategies, agree and disagree. Player l's strategy would be his first period action, $A$ or $R$, and his second period strategy, agree or disagree. For player 2 his first period strategy would be to agree or disagree. If he disagrees, we will indicate what his message will be and whether he agrees or disagrees in the next period. ${ }^{9}$ Out of equilibrium beliefs will be that if in equilibrium 1 were to send the message $A$ and $R$ was observed instead 2 will believe that 1 is of type $b$ and vice versa. Similarly, if the expected signal were $R$ and instead $A$ were observed the out of equilibrium belief will be that the individual is type $g$

Theorem 4.1: For $D \geq \tilde{D}$ there are four possible equilibria. They are
(i) $1(g):\{A$, agree $\}, 1(b):\{A$, agree $\}, 2(g):\{a g r e e\}, 2(b):\{a g r e e\}$.
(ii) $1(g):\{A$, agree $\}, 1(b):\{R$, agree $\}, 2(g):\{$ agree $\}, 2(b):\{a g r e e\}$.
(iii) $1(g):\{A$, agree $\}, 1(b):\{A$, agree $\}, 2(g) \cdot\{$ agree $\}, 2(b):\{R$, agree $\}$.
(iv) $1(g) .\{$,, agree $\}, 1(b):\{A$, agrae $\}, 2\{(\tilde{B}):\{A$, agree $\}, 2(b):\{R$, agree $\}$.

Theorem 4.2: For $\mathrm{D} \leq \tilde{\mathrm{D}}$ there are four possible equilibria which are
(i) $1(g):\{A, d i s a g r e e\}, 1(b):\{A$, agree $\}, 2(g):\{A$, agree $\}, 2(b) \cdot\{a g r e e\}$.
(ii) $1(g) \cdot\{A$, disagree $\}, 1(b):\{A$, agree $\}, 2(g):\{A$, agree $\}, 2(b) .\{R$, agree $\}$.
(iii) $1(g):\{A$, disagree $\}, 1(b):\{R$, agree $\}, 2(g):\{A$, agree $\}, 2(b):\{a g r e e\}$.
(iv) $1(g):\{R$, agree $\}, 1(b)\{A$, agree $\}, 2(g):\{A$, agree $\}$.

2(b). $\{\{\mathrm{R}$, agree\}, $\{\mathrm{A}, \mathrm{agree}\}$ \}.

A glance at table 7 below reveals that, while there are equilibria where the ideal decisions are reached, this is not always true. G3 on the other hana can reach the ideal decision in all circumstances. Thus a hierarchica: decision structure could represent an improvement over decision makirg through committees. The point though is that $G 3$ is optimal if (2) holds. I: this condition were not true and the relation between the posteriors were $t=$ be that given in (3) then G1 would be optimal. As we have indicated earlier the values of the posteriors could be interpreted as the evaluation skills of the individuals in the organization. In Gupta(1995) we show that if the costs of delay are very low the only equilibria which remain are ones whic: minimize statistical errors, when condition (2) holds. The same would be true for (3). It is possible to imagine a scenario where the designer of the organization has less information than the individuals within it and so $1=$ might be optimal to use a committee to reach decisions being secure in the knowledge that at least under some circumstances the correct decision would
be reached This would particularly be true if costs of delay were low

A comparison between the various equilibria shown in table 7 and the outcomes resulting from our discussion of the different hierarchi气ai ana oligarchical structures show that all of these are possible in a committee framework in a sense designing a hierarchy is invo:ves making a choice among the various outcomes possible in a committee and then trying to implement it through judicious use of authority relationsnips and the amount of communication allowed. As long as the individuals share the same preferences any outcome in any hierarchy exists as an outcome within a committee setting This, however, may involve delay and may only be possible for certain values of the parameters $\pi$ and $D$ and thus the choice between committees and other organizational structures is not trivial.

| events | equilibrium outcomes |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{D} \geq \tilde{\mathrm{D}}$ |  |  |  | $D \leq D$ |  |  |  |
|  | (1) | (ii) | (iii) | (iv) | (a) | (b) | (c) | ( 1.1 |
| $5 \cdot 5$ | A | A | A | A | A | A | A | A |
| $g . b$ | A | A | R | R | A | A | 4 | A |
| b. 5 | A | R | A | A | A | A | A | A |
| $b, b$ | A | R | R | R | A | R | R | R |

Table 7.

A comparison between the results of hierarchies and committees suggests that it is better to use hierarchies for decision making. Also the restriction of not more than two rounds of communication allowed is not very effective in that the committee would have in any case reached a decision in two rounds even if they were allowed unlimited rounds of discussion. The situation where this restriction has some bite is when members of the committee have
different preferences and costs of delay are $\operatorname{low}\left(D \leq \min \left\{\frac{a_{2}\left(1-\boldsymbol{\pi}_{g b}\right)}{c \pi_{g b}}\right.\right.$. $\left.\left.\frac{c \pi_{g b}}{a_{2}\left(1-\pi_{g b}\right)}\right\}\right)$. It is shown(Gupta 1995) in that case the committee could undertake more than two rounds of discussion in the form of a mixed strategy equilibrium and such discussion would be entirely fruitless in that it would be primarily a bargaining between desired outcomes. This outcome can be avoided if the committee is restricted to two rounds of discussion. The equilibria and the related outcomes are shown in theorem 43 and table $S$ (figure 1, G11). Since under the circumstances the game could take more than two periods if unlimited communication were allowed the positioning of the individuals within a committee could make a difference (figure 7) and this is investigated in Theorem 4.4 and table 9 (G12).

Theorem 4.3: There are four possible equilibria which are
(i) $1(g):\{A$, agree $\}, 1(b):\{A$, agree $\}, 2(g):\{$ agree $\}, 2(b):\{$ agree $\}$.
(ii) $1(\mathrm{~g}) \cdot\{\mathrm{A}$, agree $\}, 1(\mathrm{~b}):\{\mathrm{R}$, agree $\}, 2(\mathrm{~g}):\{$ agree $\}, 2(\mathrm{~b}):\{$ agree $\}$
(iii) $1(g):\{A$, agree $\}, 1(b):\{A$, agree $\}, 2(g):\{$ agree $\}, 2(b) \cdot\{R$, disagree $\}$.
(iv) $1(g):\{A$, agree $\}, 1(b):\{R$, agree $\}, 2(g):\{$ agree $\}, 2(b):\{R$, disagree $\}$.

| events | equilibrium outcomes |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | (i) | (ii) | (iii) | (iv) |
|  | A | A | A | A |
|  | A | A | R | R |
| $b, g$ | A | R | A | R |
| $b, b$ | A | R | R | R |

Table 8.

The equilibrium outcomes are similar to $G 8$ and the restriction does not seem to do undue damage to desirable outcomes. There is one equilibrium which a designer with $a=a_{2}$ would consider ideal (equilibrium (iv)) and given the


```
off However, tris resuit is sensi*ive to tne rnoree of ine inaıviaual who
starts the game if Inalvidual 2 startstr gite 7) tnen the possible
equi!:ar:am would se a|ferert as lo shumn in -heorem f f a a a iad_e a
```

Theorem 44 There are four possible equllior.a wnim are (1) $2(g)\{R$, agree, $2(b)\{R$, agree $\}$. : (g, lagree\} $1(b)$ \{agree \} (11) $2(g)\{A$, agree $\}, 2(b)\{R$, agree $\}, 1(g)$ \{agree, $1|b|$ \{agree $\}$ (11: $2(g)\{R$, agree', 2'b) \{R, agreer, $1(g)$ ia disagree', $i(b)\{a g r e e\}$ (1ッ) $2(g)\{A$, agree $\} 2(b)\{R$, agree\}. $1: g)\{4$ aisagree\}, $1(b)$ \{agree\}


## Figure 7

In this case equilibrium iv produces outcomes which a designer with $a=a_{1}$ would find appealing. This outcome would result if the individuals are patient If not it is possible (equilıbrium (i)) that the project is always rejected We have seen earlier that, in the same circumstances, if the order of play were to be reversed there would be equilibria which would be appealing to a des.gner witn preferences $a_{2}$


#### Abstract

Thus our earlier statement about information requirements about setting up a committee would have to suitably altered. Now it seems that if the committee members are biased and have a great deal of patience then an effort to cut down on the length of deliberation would have to attentive to the extensive form of the game. Earlier we said that a committee framework supports all outcomes possible with other organizational structures and that statement remains valid as a comparison between the outcomes of $G 7, G 8$ and G11 shows In some cases the ideal outcomes can be achreved but that depends on the value of $\pi$ and $D$ Particularly if $D$ is high then an oligarchical structure could be optimal.


## 5. Conclusion

Our efforts in this paper have been geared towards investigating the efficacy of hierarchies in decision making. We have discovered that given good information on the structure of information it is possible to design a hierarchy which would suit a particular designer's preferences. This is more easily done if the individuals in an organization have the same preferences. If individuals are biased then it would take more time and the information requirements increase and particular attention has to be paid to the position of individuals in a hierarchical structure.

A committee on the other hand has less information requirements and would be suitable if the individuals in the organization had the same preferences. If individuals are biased then the extensive form again becomes important if individuals are patient and an effort is made to curtail the deliberation process by reducing the time allotted for reaching decisions.

The question as to which organ. ationi: si:uriure i the ses depends crucially on whether the individuais in the team share ine same preferences over outcomes. If they do then one can design a hierarch; which achieves the ideal outcomes for all vaiues of $\pi$ a committee can do equally well if the cost of delay is low. This result seems to fit witt recent concerns about motivating teams properly As "The Economist" notes ' : typica. mistake is the failure to set clear objectives Another is to introduce teams without changing the firm's pattern of appraisal from an individuai to a collective system " The use of teams in manufacturing whereby local problems are solved jointly by the members could serve as an example of the imporiance of low costs of delay and shared preferences If the individuals are biased then it becomes more difficult to implement desired outcomes. If the values of $\pi$ and D are not right this becomes impossible and oligarchies then become suitable for implementing outcomes. The main benefit is that oligarchies dvoid delay

It could be argued that since our committee comprises of only two individuals it is not possible to invesiteate other torms of committee decision making like voting. Such oriticism ruuld be vaiid except that an important feature of decision making through committees is a requirement of some form of consensus and this is saptured in our model. We hope to extend our model to deal with more than two individuals in future research.

## APPENDIX A

Proof of Theorem 3.1
(i) Consider $1(g)$. The payoff from $A$ is $a\left(1-\pi_{g g}\right) \pi+a\left(1-\pi_{g b}\right)(1-\pi)$. This is the statistical error of wrongly accepting a project. The payoff from $R$ is $c \pi_{g q} \pi+c \pi_{g b}(1-\pi)$. Thus $A$ will be optimal in minimizing statistical error if
$a\left(1-\pi_{g g}\right) \pi+a\left(1-\pi_{g b}\right)(1-\pi) \leq c \pi_{g g} \pi+c \pi_{g b}(1-\pi)$, but
$a\left(1-\pi_{g g}\right) \leq c \pi_{g g}$ and $a\left(1-\pi_{g b}\right) \leq c \pi_{g b} \operatorname{since} \pi_{g g}>\pi_{g b}>\frac{a}{a+c}>\pi_{b b}$, $s 0$ the condition is satisfied. For $l(b)$ A is optimal if
$\mathrm{a}\left(1-\pi_{\mathrm{qb}}\right) \pi+\mathrm{a}\left(1-\pi_{\mathrm{bb}}\right)(1-\pi) \leq c \pi_{\mathrm{gb}} \pi+c \pi_{b b}(1-\pi)$
which means $\pi \geq \frac{\frac{a}{a+c}-\pi_{b b}}{\pi_{g b}-\pi_{b b}}$
$2(g)$ is in a similar position as $l(g)$. The same is true for $2(b)$ and $l(b)$.
(ii) and (iii) can be shown in a similar manner.

Proof of Theorem 3.2
(i) The players' first period strategy is the same as in Theorem 3.1. In the second period if the sequence of messages $\{A, R\}$ is observed then player: would believe that player 2 is type $b$. Then saying $R$ and ending the game is optimal given D. Similarly, if player 2 finds himself contemplating an action after the sequence $\{A, R, A\}$ then by his belief player 1 has observed the signal $g$ and so he should play $A$.
(ii) The situation is similar to (i) except that $\boldsymbol{\pi} \leq \tilde{\pi}$.
(iii) From before the strategies in the second period are optimal given the beliefs. So we will concentrate on strategies in the first period. For $1(g)$ the payoff from $A$ is
$a\left(1-\pi_{g g}\right) \pi+\operatorname{Dc}_{g b}(1-\pi)$
while that from $R$ is
$c \pi_{g q} \pi+c \pi_{g b}(1-\pi)$.
So A is optimal if
$a\left(1-\pi_{g g}\right) \pi+D c \pi_{g b}(1-\pi) \leq c \pi_{a q} \pi+c \pi_{c h}(1-\pi)$
or $\pi \geq \pi_{1}$ The condition for $l(b)$ is, similarly, $\pi \geq \pi_{2}$ for $2(g)$ A is a dominant strategy For $2(b)$ the payofi from $A$ is
$a\left(1-\pi_{g b}\right) \pi+a\left(1-\pi_{b b}\right)(1-\pi)$
while that from $R$ is
$D\left\{c \pi_{g b} \pi+c \pi_{b b}(1-\pi)\right\}$.
Then $R$ is optimal if $\pi \leq \pi_{3}$.
The proofs of rest of the results are similar and are not shown.

APPENDIX B

We need the incentive compatibility conditions which will make the two players reveal their signals truthfully. Beginning with player 1 , it must be the case that he says $g$ rather than $b$ when he receives $g$. The condition which assures this is

$$
\begin{aligned}
& \pi a_{1}\left(1-\pi_{g g}\right) x_{1}+\pi c \pi_{g g}\left(1-x_{1}\right)+(1-\pi) a_{1}\left(1-\pi_{g b}\right) x_{2}+(1-\pi) c \pi_{g b}\left(1-x_{2}\right) \\
& \leq \pi a_{1}\left(1-\pi_{g g}\right) x_{3}+\pi c \pi_{g g}\left(1-x_{3}\right)+(1-\pi) a_{1}\left(1-\pi_{g b}\right) x_{t}+(1-\pi) c \pi_{g b}\left(1-x_{f}\right)
\end{aligned}
$$

or,
$\pi x_{1}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g g}\right\}+(1-\pi) x_{2}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g b}\right\}$
$-\pi x_{3}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g g}\right\}-(1-\pi) x_{i}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g b}\right\} \leq 0$

The conditions for player 1 to say $b$ when he receives $b$ is
$-\pi x_{1}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g b}\right\}-(1-\pi) x_{2}\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\}$
$+\pi x_{3}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g b}\right\}+(1-\pi) x_{4}\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\} \leq 0$.

The two corresponding conditions for player 2 are
$\pi x_{1}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g g}\right\}-\pi x_{2}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g q}\right\}$
$+(1-\pi) x_{3}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}-(1-\pi) x_{t}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\} \leq 0$.
$-\pi \times_{1}\left\{a_{2}-\left(a_{2}+c\right) \pi_{q b}\right\}+\pi x_{2}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}$
$-(1-\pi) x_{3}\left\{a_{2}-\left(a_{2}+c\right) \pi_{b b}\right\}+(1-\pi) x_{1}\left\{a_{2}-\left(a_{2}+c\right) \pi_{b b}\right\} \leq 0$
Thus the principal's problem is to
minimize $\pi^{2} x_{1}\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g q}\right\}-\pi(1-\cdot)\left(x_{2}+x_{3}\right)\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi \pi_{i} ;\right.$

$$
\begin{aligned}
& (1-\pi)^{2} x_{4}\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{b b}\right\}+ \\
& \pi^{2} c_{0} \pi_{\mathrm{gq}}+2 \pi(1-\pi) c_{0} \pi_{\mathrm{gb}}+(1-\pi)^{2} c_{0} \pi_{\mathrm{bb}}
\end{aligned}
$$

subject to
$\pi \times,\left\{\alpha_{1}-\left(a_{1}+c\right) \pi_{g q}\right\}+(1-\pi) x_{2}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g b}\right\}$
$-\pi x_{3}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g \mathrm{gq}}\right\}-(1-\pi) x_{4}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g b}\right\} \leq 0$
$-\pi x_{1}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g b}\right\}-(1-\pi) x_{2}\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\}$
$+\pi x_{3}\left\{a_{1}-\left(a_{1}+c\right) \pi_{q b}\right\}+(1-\pi) x_{4}\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\} \leq 0$
$\pi<_{1}\left\{d_{2}-\left(a_{2}+c\right) \pi_{g g}\right\}-\pi x_{2}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g g}\right\}$
$+(1-\pi) x_{3}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}-(1-\pi) x_{4}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\} \leq 0$
$-\pi \kappa_{1}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}+\pi x_{2}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}$
$-(1-\pi) x_{3}\left\{a_{2}-\left(a_{2}+c\right) \pi_{b b}\right\}+(1-\pi) x_{4}\left\{a_{2}-\left(a_{2}+c\right) \pi_{b b}\right\} \leq 0$.

The unconstrained optimum for (i) would be to set $x_{1}=x_{2}=x_{3}=1$ and $x_{4}=$
0 . However, these values do not satisfy the constraints We see chat (7), (3) and (10) are satisfied while (9) is not Thus (9) must be binding Writing (9) with equality,
$\pi x_{1}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g q}\right\}-\pi x_{2}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g g}\right\}$
$+(1-\pi) x_{3}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}-(1-\pi) x_{4}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}=0$

We note that the first term is negative so that an increase in $x_{1}$ will allow us to increase $x_{2}$ and $x_{3}$ and decrease $x_{1}$. So $x_{1}=1$ must be a part of the solution. The rest of the solution will depend on the second and third terms
of the objective function If the absolute value of the term accompanying $x_{2}$ and $x_{3}$ is higher than the one accompanying $x_{4}$ it would be optimal to increase $x_{2}$ and $x_{3}$ at the expense of decreasing $x_{4}$. Thus, if
$\left|\pi\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g b}\right\}\right|<\left|(1-\pi)\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{b b}\right\}\right|$
the solution would be $x_{1}=x_{2}=x_{3}=x_{t}=1$.

If the inequality in (11) is reversed then $x_{4}=0$ Substituting the values of $x_{1}$ and $x_{t}$ in (9) we get
$\pi\left\{a_{2}-\left(a_{2}+c\right) \pi_{g q}\right\}-\pi x_{2}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g g}\right\}$
$-(1-\pi) x_{3}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}=0$

The principal should, from has objective function, try to get the largest combined value of $x_{1}$ and $x_{2}$ as possible. The solition should now depend on the absolute value of the terms accompanying $x_{2}$ and $x_{3}$ in (12). Thus, if

$$
\begin{equation*}
\left|\pi\left\{a_{2}-\left(a_{2}+c\right) \pi_{g 9}\right\}\right|<\left|(1-\pi)\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}\right| \tag{13}
\end{equation*}
$$

it would be optimal to set $x_{2}=1$ and then $x_{3}=0$. If the inequality in (13) is reversed then we should set $x_{3}=1$ and $f$ ind the value of $x_{2}$ from (12)

Thus the complete solution to the optimization problem is
$x_{1}=x_{2}=x_{3}=x_{t}=1$ if (11) holds,
$x_{1}=1, x_{2}=1, x_{3}=0, x_{4}=0$ if (13) holds but not (11) and
$x_{1}=1, x_{3}=1, x_{4}=0$ and $x_{2}=1+\frac{(1-\pi)\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}}{\pi\left\{a_{2}-\left(a_{2}+c\right) \pi_{g g}\right\}}$ if neither
nor (13) holds

From symmetry we conclude that the corresponding solution for (il) not nolaing w.ii be
$x_{1}=x_{2}=x_{3}=x_{4}=0{ }_{1} f$
$\left|(1-\pi)\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g b}\right\}\right|>\mid \pi\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{G G}\right\}$
$x_{1}=1, x_{2}=0, x_{3}=1, x_{4}=0$
If $\left|(1-\pi)\left\{a_{0}-\left(a_{0}+c_{0}\right) \pi_{g b}\right\}\right|>\left|\pi\left\{a_{0}-\left(a_{0}+c_{0}\right\} \pi_{G g}\right\}\right|$ and
$\left|(1-\pi)\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\}\right|<\mid \pi\left\{a_{1}-\left(a_{1}+c\right) \pi_{100}\right\}$,
$x_{1}=1, x_{3}=0, x_{4}=0$ and $x_{2}=-\frac{\pi\left\{a:-\left(a_{1}+c\right) \pi_{g b}\right\}}{(1-\pi)\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\}}$ if neither of the above two conditions hold.

In this rase the constraint that will be anding is (S) ard the pr ncipal would want to set $x_{1}=1$ and $x_{2}=x_{3}=x_{1}=0$ wien will satisfy all the constraints except (S)

The final optimization we will consider is the sum of the two players' utilities. Player 1 's payoff is given by
$\pi^{2} x_{1}\left\{a_{1}-\left(a_{1}+c\right) \pi_{g g}\right\}+\pi(1-\pi)\left(x_{2}+x_{3}\right)\left\{a_{1}-\left(a_{1}+c\right) \pi_{g b}\right\}+$ $(1-\pi)^{2} x_{f}\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\}+\pi^{2} c \pi_{g q}+2 \pi(1-\pi) c \pi_{G b}+(1-\pi)^{2} c \pi_{b b}$
and similarly player 2 's payoff is
$\pi^{2} x_{1}\left\{a_{2}-\left(a_{2}+c\right) \pi_{g g}\right\}+\pi(1-\pi)\left(x_{2}+x_{3}\right)\left\{a_{2}-\left(a_{2}-c\right) \pi_{g b}\right\}+$
$(1-\pi)^{2} x_{4}\left\{a_{2}-\left(a_{2}+c\right) \pi_{b b}\right\}+\pi^{2} c \pi_{g g}+2 \pi(1-\pi) c \pi_{G b}+(1-\pi)^{2} c \pi_{b b}$

The sum of their utilities will be

$$
\begin{aligned}
& \pi^{2} x_{1}\left\{\left\{a_{1}-\left(a_{1}+c\right) \pi_{g g}\right\}+\left\{a_{2}-\left(a_{2}+c\right) \pi_{g g}\right\}\right\}+ \\
& \pi(1-\pi)\left(x_{2}+x_{3}\right)\left\{\left\{a_{1}-\left(a_{1}+c\right) \pi_{g b}\right\}+\left\{a_{2}-\left(a_{2}+c\right) \pi_{g b}\right\}\right]+ \\
& (1-\pi)^{2} x_{4}\left[\left\{a_{1}-\left(a_{1}+c\right) \pi_{b b}\right\}+\left\{a_{2}-\left(a_{2}+c\right) \pi_{b b}\right\}\right]+ \\
& 2 \pi^{2} c \pi_{g g}+4 \pi(1-\pi) c \pi_{g b}+2(1-\pi)^{2} c \pi_{b b}
\end{aligned}
$$

From (3) the first term is always negative and the third term is always negative Thus the solution will depend on the term within the square brackets in the second term If it is negative then the first set of solutions will hold, otherwise the second set of solutions will be applicable.

1 Koh does consider the question of implementation of optima: cut-off points His investigations reveal that it is generally not possible to induce agents to choose the optimai cut-off points by means of a compensation scheme based on the success or failure of projects.

2 There could be a problem with the cost of rejecting a successfal project The question is how would the indiviauals know that they have rejected a successful project and, thus, feel the cost However, projects rejected by the organization would be available for acceptance by other firms an organizarion which consistently rejects successful projects cannot remain in business for long. Thus, c could be interpreted as the induced cost of prospect:ve unemployment.
3. We are assuming that the individuals are risk neutral it is possible that the structure of organizations could affect the riskiness of the portfolic of firms. However, that will not be investigated here.

4 Sah and Stiglitz emphasize that their analysis can be viewed from two different perspectives. First, as an examination of the effect of internal structure of organizations on the portfolio of projects accepted, and, second as an examination of the relative virtues of markets and central planning Their assumption that projects if rejected by one polyarchy are then avallable to others is troubling. Unless the number of polyarrhies is large there exists the possibility of strategic acceptance of projects Their analysis, however, neglects this possibility
5. Katzner (1992) calls this the span of control

6 Radner (1992, 1993), Radner and Van Zandt(1901) consider the effect of the structure of organizations and length of time required to perform tasks
7. In Theorem 3.2 we can get different sequential equilibria from the ones shown by assigning different beliefs. For example in case of the sequence of observations $\{A, R\}$ player $l$ may believe that he is facing type $g$ and say $A$ for the appropriate value of $D$. However. this belief would not satisfy the Cho and Kreps (1987) concept of the intuitive criterion. For a detailed discussion see Gupta (1995).
8. In equilibrium (iv) the outcome $R$ is achieved in the event ( $g, b$ ) with delay. If we allowed the principal to impose the same cost on the two players if player 1 reported $g$ and player 2 reported $b$ we would be able to achieve the same outcomes as in the hierarchy. In the appendix we note that in attempting to set $x_{1}=1, x_{2}=x_{3}=x_{4}=0$. the principal finds out that (8) would not be satisfied. However if we include the cost of delay then (8) would be satisfied for some values of the parameters if we substitute this particular solution.
9. For a sequential equilibrium we should specify strategies at all nodes, even those that will not be reached if the equilibrium strategies are played. Given the out of equilibrium beliefs it is quite easy to see what these should be. These are also discussed in Gupta (1995).

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