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Dynamic Pricing under Social Learning with Strategic Consumers

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A monopolist uses prices as an instrument to influence consumers' belief about the unknown quality of its product. Consumers observe prices and sales in earlier periods to learn about the product. Every period they decide whether to consume the product or to wait for a lower price in future. We solve for the optimal price strategy of a monopolist. We show that for certain range of beliefs prices increase over the period of time. Per period profits increase over the period of time. We find that the firm encourages social learning for a greater range of beliefs and has greater expected revenue when it faces consumers that can delay their purchase decision versus when they can't.

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1 Introduction

When the quality of the product is uncertain, firms may use price to influence future perception and sales of the product. Literature finds that in such a scenario optimal pricing strategy generates a super-martingale price sequence, i.e. on an average prices decrease with time.[1] Anticipating future price cuts consumers frequently delay their purchase decisions. According to SmartMoney magazine, there is constantly a "cat- and mouse" game between retailers, who hope to charge full price for everything, and shoppers, who wait for a price reduction.[10] An ex Chief Executive Officer of Best Buy describes this behavior of consumers as "devils" who wait for markdowns and respond to promotions, and apply for rebates. According to their estimates twenty percent of its consumers are these "devils". Given this complex consumer behavior, our paper address the challenging issue of optimal pricing strategy of a firm selling a new durable good when consumers can delay their purchase decisions.

In our model neither the seller nor consumers know the true quality of the product. This means that the pricing strategy determines the flow of information to consumers as well as to the monopolist. We analyze optimal price movement in such an environment. Next, we analyze the impact of consumer's option to wait on revenue of the firm.

We analyze a two period model, where every period a new consumer enters the market. At the beginning of the first period the monopolist decides the price of the product for that period. Period 1 consumer inspects the product and receives an imperfect but independent private signal about the quality of the product. Period 1 consumer updates his belief about the quality and decides whether to purchase the product in the first period or to wait. At the beginning of the second period, the monopolist and the second period consumer observe first period price and purchase decision and update their belief. Next, monopolist decides second price. Period 2 consumer inspects the product and updates his belief about the product. All 'active' consumers, consumers that have not yet purchased the product decide whether to consume the product or not in the second period.

Our findings concern the structure of the optimal pricing policy. The optimal pricing strategy of the monopolist is a threshold policy and depends upon the number of 'active' consumers in that period. We find that for very low beliefs the monopolist finds it disadvantageous to introduce the product to any type of consumer. However, the product is sold to all type of consumers when the beliefs are very high.

Our main result is that the progression of prices over time depends upon the initial prior. Unlike the non strategic consumer case, the price path is neither super

[1] Bose et al [4] and Bhalla [2]

or sub martingale. For a range of beliefs, price path sees a decline over time. The reason is informational. If there is a purchase at high price, it conveys to later consumers that the product is more likely to have been of high quality. On the other hand, if there is no purchase then it indicates to later consumers that the product is more likely to be of low quality. Charging a low price is safe but does not increase future payoffs. However, high prices are risky but can improve future profits. For a range of beliefs, the monopolist finds it profitable to take this bet.

On the other hand, there exists a range of beliefs such that the optimal price increases over time- that is, price signal form a sub-martingale. To induce consumers to purchase the product in the first period at a high price, the monopolist must compensate them for their opportunity cost from waiting. When consumers have the option to wait their opportunity cost is higher. Thus, price on an average increases in the next period. We also find that on an average per period profits and number of consumptions increase over time. On the equilibrium path only the Low type fresh consumer delays his consumption. Thus, delay is informative of a low signal and would be discouraged by the firm.

Next, we compare the optimal pricing strategy of the firm to the scenario when consumers do not have an option to wait. We find that the firm encourages learning more when consumers have the option to wait than when they do not. In the strategic consumer scenario, a consumer is in the market in the second period even if it does not purchase the product in the first period. Thus, the expected loss (of not making a sale to the first period consumer) is lower in the strategic consumer case. Hence, the firm encourages learning more when consumers have an option to wait than when they do not.

We also find that total expected revenue is more for a monopolist when it faces a non strategic consumer for most ranges of beliefs.

Related Literature

This paper contributes to two strands of literature : (i) dynamic pricing strategy under social learning and (ii) pricing with strategic consumers.

Economists have devoted considerable attention to the role of prices as instruments of information transmission under uncertainty. Milgrom and Roberts (1986) [12] and Bergemann and Valimaki (1995, 2000)[16, 17] find the optimal pricing strategy of an experience good with repeat purchase. Bose et al. (2008) [4] look at the dynamic pricing problem of a durable inspection good when consumers learn from each other but are not strategic. They find that the optimal pricing policy is a super-martingale. Unlike Bose et al (2008) we analyze the pricing problem of the an inspection good when it faces strategic consumers and find that the price trend depends upon the initial belief of the product. For low initial beliefs prices are a super martingale. However, for high initial beliefs the prices are a sub martingale.

The literature on pricing problem with strategic consumer behavior pioneered by Coase [7] and Stokey [13, 14] generally follows the rational expectations approach where consumers anticipate future price changes and adjust their purchase timing in response. This strand of literature focusses on differences in discount rates and preferences as rationales for the optimal pricing. Levin et al [11] analyze the pricing problem of differentiated perishable goods under different degrees of competition. Cachon and Swinney [5] solve the optimal order quantity and price of the product with strategic consumers. Our paper differs in that we solve the dynamic pricing strategy problem of a monopolist which is also uncertain about the quality of the product. Chen[6] and Villas-Boas [15] look at the optimal targeted pricing strategy for goods with repeat purchase when consumers are strategic. They find that the optimal pricing policy is a super-martingale and that strategic consumer reduces monopoly power and profits. Our paper differs from them in multiple ways. We look at the pricing problem of a monopolist selling a durable good whose quality is learnt over the period of time. Consumers value of the product is endogenous and we also do not allow for repeat purchase. We find that the price trend depends upon the initial belief. Surprisingly, we also find that the expected revenue is higher in the strategic consumer case than otherwise. Gunay (2008)[8] is a signalling problem of the firm that knows the quality of the product and faces strategic consumer. In contrast our paper looks at the pricing problem of a new good, quality of which is unknown even to the monopolist.

2 Model

A risk neutral monopoly seller sells a new durable good of unknown value, ω over two periods. The unknown value of the product, ω can either be High, i.e. $\omega = H$ or Low, $\omega = L$. Each period one consumer enters the market. Both the seller and consumers are unaware of the true value of the product.[2] At the beginning of the game, the common prior about the state of the world being High, i.e. $\omega = H$ is q_1.

At the beginning of the first period monopolist decides first period price, p_1. Period 1 consumer enters the market and inspects the product. Each consumer while inspecting the product in the period when he enters the market receives an independent private signal, $s \in \{h, l\}$ about the quality of the product. The signal

[2] Since the product is new, its characteristics are known only to the firm. Similarly, only consumers are aware of their preferences. The uncertain value can be interpreted as a match between characteristics of the product and consumer preferences. We assume that all consumers have the same preferences.

received by a consumer remains the same throughout his life time. These signals are costless, independent and privately observed by each agent. However, it is common knowledge that the signals are distributed independently given the state of the world according to Table 1.

Table 1: Signal structure

$P(s\|\omega)$	h	l
H	λ	$1-\lambda$
L	$1-\lambda$	λ

λ can be interpreted as the precision or accuracy of the signal and is assumed to be greater than half. This precision or accuracy is assumed to be the same for all consumers. Given the signal received, period 1 consumer further updates his/her belief about the product. Upon seeing first period price, p_1 first period consumer decides whether to consume the product at price p_1 or to wait. At the beginning of the second period, the seller and period 2 consumer observe first period price and purchase decision. They use this information to update the common public belief. Period 2 consumer inspects the product and receives his independent private signal and updates his belief about the product. All 'active' consumers, consumers who have not yet purchased the product decide whether to consume the product or not.

Let $c_t = c_t(p_t)$ be the number of purchases at time period t, $\forall t \in \{1, 2\}$. We denote the action of a consumer with signal $s \in \{h, l\}$ at time period t with τ_t^s, which takes value 1 upon a purchase and zero otherwise. The product is a durable good and the decision to purchase the product is taken once and is irreversible. Thus, if $\tau_1^s = 1$ for $s \in \{h, l\}$ then $\tau_2^s = 1$. The realized utility of a consumer at time period t depends upon the underlying state of the world, ω and the action of the consumer with signal $s \in \{h, l\}$, τ_t^s according to Table 2.

Table 2: Realized utility of the consumer

$U(\tau_t^s, \omega)$	$\omega = H$	$\omega = L$
$\tau_t^s = 1$	1	-1
$\tau_t^s = 0$	0	0

Consuming the product gives a positive realized utility when the underlying state of the world is High versus when it is not. The outside option of each consumer is zero. The payoff of a consumer that received a signal $s \in \{h, l\}$ is a discounted sum of utility, $\sum_{t=1}^{2} \beta^{t-1} EU(\tau_t^s)$ where β is the discount factor.

The marginal cost of production and fixed cost of selling the product to consumers are assumed to be zero. The seller's payoff is the discounted sum of revenue, $\sum_{t=1}^{2} \beta^{t-1} p_t c_t$ where β is the discount factor. We find pure strategy Perfect Bayesian Equilibrium (PBE) for this game between the monopoly seller and consumers. The dynamic pricing strategy of the seller and consumption decision of the consumers is solved. At the beginning of the second period first period price, p_1 and purchase decision, c_1 determines the public belief about the value of the product, $q_2 = Pr(\omega = H|p_1, c_1)$. The common public belief, q_t and number of 'active' consumers, a_t at any point time t are the state variables of the seller's dynamic problem. The associated value function is denoted by,

$$V_1(q_1, a_1) = Max_{p_1} p_1 E[c_1(p_1)] + \beta EV_2(q_2, a_2)$$

where the first term is the expected current payoff from selling the product at price p_1. The second term is the expected payoff that the firm receives at time period 2 given that the product was sold at price p_2 at time period 2. Similarly, the monopolist solves the following problem in the second period:

$$V_2(q_2, a_2) = Max_{p_2} p_2 E[c_2(p_2)]$$

3 Equilibrium Analysis

The first part of the section discusses the updating rule of common public belief about the quality of the product.

At the beginning of second period, there is only one active consumer- period 1 consumer. Once the fresh consumer inspects the product and receives an independent private signal they update their prior about the unknown value of the product.[3]

Let the updated posterior belief of the state for a consumer that receives signal, $s \in \{h, l\}$ in the first time period be $f_s(q_1) = Pr(\omega = H|s, q_1)$. By Bayes rule

$$f_h(q_1) = \frac{\lambda q_1}{\lambda q_1 + (1-\lambda)(1-q_1)}$$

$$f_l(q_1) = \frac{(1-\lambda) q_1}{\lambda(1-q_1) + (1-\lambda) q_1}$$

[3]This updated belief is independent and private to each consumer who inspects the product.

Notice that $f_h(q_1)$ and $f_l(q_1)$ are increasing, concave and convex functions in q_1 respectively. The seller can sell the product at either of the two prices: *High price*, $P_H(q_1) = 2f_h(q_1)-1$, the expected utility of the High type Period 1 consumer; or *Low price*, $P_L(q_1) = 2f_l(q_1) - 1$, the expected utility of the Low type Period 1 consumer. Any price other than these two prices would generate a lower revenue.

The common public belief at the beginning of the second time period, q_2 is determined by observing public history, $h_2 = (p_1, c_1)$ which is observed by the firm and second period consumer. The updated belief when there are no delayed consumers at the beginning of the second time period can be written as:

$$q_2 = \begin{cases} q_1; & p_1 \leq P_L(q_1) \\ \frac{q_1\lambda^{c_1}(1-\lambda)^{1-c_1}}{q_1\lambda^{c_1}(1-\lambda)^{1-c_1}+(1-q_1)(1-\lambda)^{c_1}\lambda^{1-c_1}}; & p_1 \in (P_L(q_1), P_H(q_1)], \\ q_1; & p_1 > P_H(q_1) \end{cases}$$

For any price less than equal to the Low price, $P_L(q_1)$ there would always be a purchase. Since, the price is always lower than the expected utility of all types of first period consumer consume the product. Since no additional information is obtained, the updated belief is $q_2 = q_1$.

For price, $p_1 \in (P_L(q_1), P_H(q_1)]$ there would be a purchase only period one consumer that receives a high signal. Thus, the updated belief can be written as

$$q_2 = \frac{q_1\lambda^{c_1}(1-\lambda)^{1-c_1}}{q_1\lambda^{c_1}(1-\lambda)^{1-c_1} + (1-q_1)(1-\lambda)^{c_1}\lambda^{1-c_1}}$$

For price greater than the High price, $P_H(q_1)$, there would be no purchase as the price is higher than the expected utility of all types of period 1 consumer. Since no additional information is obtained, $q_2 = q_1$.

3.1 Last Period Pricing Strategy

The last period pricing strategy depends upon the updated belief about the product, q_2, and the number of 'active' consumers, a_2 at the beginning of second period. The following lemma details the last period pricing strategy.

Lemma 1. ***Optimal Last Period Pricing Strategy***

- *If there is only one active consumer at the beginning of the second period, i.e.* $a_2 = 1$ *then the optimal second period pricing strategy is the following*
  - *For sufficiently low beliefs, the seller does not sell.*

- *For intermediate beliefs, the seller sells the product such at a price such that there is consumption only if the consumer received a high signal.*
- *For sufficiently high beliefs, the seller sells the product such that there is always a consumption.*

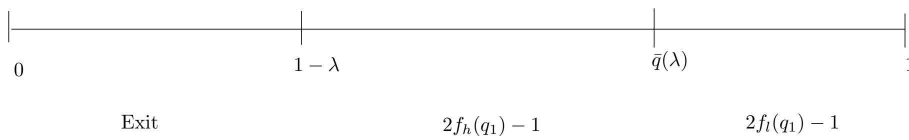

Figure 1: Last Period Pricing, $d_2 = 0$

- *If there are two 'active' consumers at the beginning of the second period, i.e. $a_2 = 2$ then the optimal second period pricing strategy is the following*

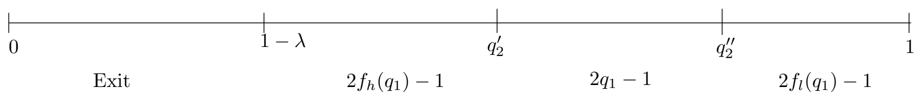

Figure 2: Last Period Pricing, $d_2 = 1$

- *For sufficiently low beliefs, the seller does not sell.*
- *For larger beliefs, the seller sells the product at a price such that there is a consumption only of the second period consumer received a High signal.*
- *For intermediate beliefs, the seller sells the product such that period one consumer always consumes the product and period 2 consumer consumes the product only if it received a High signal.*
- *For sufficiently high beliefs, the seller sells the product at the Low price such that there is always a consumption.*

Proof. See Appendix A □

If there is only one active consumer at the beginning of the second period, then this consumer (Period 2 consumer) could either be of High or Low type. Thus, the monopolist could charge either of two different prices- High Price, expected utility of the High type consumer, $P^H(q_2) = 2f_h(q_2) - 1$ or Low price, expected utility of the Low type consumer, $P^L(q_2) = 2f_l(q_2) - 1$. For second period common prior less than $1 - \lambda$, the expected utility of the High and Low type consumer is negative. Thus, for this range of beliefs, the seller does not sell its product to any consumer. When second period common prior is between $1-\lambda$ and λ, only the High type consumer has positive expected utility. Thus, the product is sold at a price equivalent to the expected utility of the High type consumer. When belief is greater than λ, the expected utility of both the High and Low type consumer is positive. Selling the product at the High price generates a higher per sale revenue but the probability of sale is less than one. Low price guarantees a sale but a lower per sale revenue. The difference between the two prices decreases with increase in belief.[4] Thus, the benefit of selling the product at the High price decreases as belief increases. Hence, there exists a threshold, $\bar{q}(\lambda)$ such that for beliefs above $\bar{q}(\lambda)$ the seller would sell to all type of Period 2 consumer at the Low price and for beliefs less than $\bar{q}(\lambda)$ the product is sold at a High price such that only the High type Period 2 consumer consumes it.

A similar analysis gives us the above mentioned result for the case when there are two active consumers.

The following lemma describes the first period pricing strategy.

Proposition 1. ***First Period Pricing Strategy***

Optimal first period pricing strategy of the monopolist is a threshold policy as follows.

- *For low beliefs the product is not introduced in the market*
- *For intermediate beliefs the product is introduced at a price such that only High type Period 1 consumer consumes the product.*
- *For extreme beliefs the product is introduced such that there is always a consumption.*

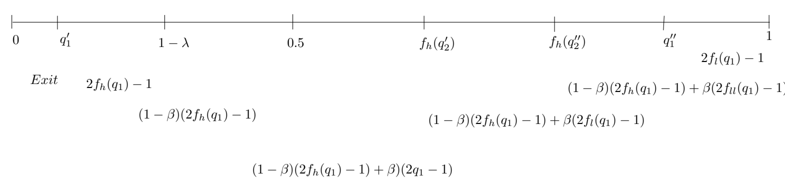

Figure 3: Last Period Pricing, Non strategic consumers

[4] $f_h(q), f_l(q)$ are concave and convex increasing functions respectively. For $q=0.5$, $\frac{\partial f_h(q)}{\partial q} = \frac{\partial f_l(q)}{\partial q}$. Thus, $\forall q \geq 0.5$, $\frac{\partial f_h(q)}{\partial q} \leq \frac{\partial f_l(q)}{\partial q}$. Therefore, $\forall q \geq 0.5, \frac{\partial P_H(q_t)}{\partial q_t} \leq \frac{\partial P_L(q_t)}{\partial q_t}$. Or $\frac{\partial[P_H(q_t)-P_L(q_t)]}{\partial q_t} \leq 0 \forall q \geq q(\bar{\lambda}) > 0.5$

Proof. Appendix B □

In the first period the monopolist faces only one active consumer. The firm can charge a price such that either there is always a consumption or that period 1 consumer consumes the product only if it receives a High signal. We find that the optimal first period pricing policy is a threshold policy such that for low beliefs the product is not sold to any type of Period 1 consumer in the market. For intermediate beliefs the product is introduced to only High type Period 1 consumer. For very high beliefs the product is sold to all types of Period 1 consumer.

We find that for a range of beliefs, the seller introduces the product to the High type consumer at a price below marginal cost. The price is below marginal cost but high enough such that only the High type consumer consumes the product. Any consumption at this price implies a high signal and favorable updating of next period belief. In such a scenario, for beliefs greater than q_1' current losses are more than compensated by the positive future expected profits. Introducing the product at a price such that all types of Period 1 consumer consume the product is not informative about the quality of the product. If the monopolist wants to improve the belief about his product then he must introduce the product at a price such that the consumer's private signal is revealed. This is done only when the product is launched at the High price. Subsequently, as the belief is improved the benefit from improving next period beliefs decreases and the product is introduced in order to capture the entire market at a low price. Thus, there exists a cutoff level of belief q_1'', such that for beliefs greater than q_1'' the seller sells the product at the High price and for beliefs lower than q_1'' the product is sold at the Low price.

The price charged to the High type consumer is decreasing in period 1 common public belief. The price charged to the High type fresh consumer compensates the consumer for the opportunity cost of waiting. As the belief about the quality of the product increases the opportunity cost of waiting increases. Hence, the price charged decreases with belief.

Corollary 1. *On the equilibrium path, only the consumer that receives a Low signal delays his consumption.*

The optimal price sequence is such that either all types of period 1 consumer consume the product or such that only the high type period 1 consumer consumes it. Thus, there will be delay only when the product is priced at the High price and period 1 consumer received a low signal. Delay is informative of Low signal received ad would be discouraged by the firm.

4 Trend in Prices

This subsection discusses the price path in equilibrium. Bose et al (2008) [4] and Bhalla [2] find that when consumers are non strategic prices, of a new inspection durable good decreases on an average. However, we find that this result no longer holds true when consumers can choose when to consume. The following proposition describes the price trend. We find that the price trend depends upon the belief of the product.

Proposition 2. ***Prices are neither Super-Martingale nor Sub-Martingale***
If $\beta > 2/3$, prices on an average decrease over time for beliefs between $q_1 \in [q_1', 1-\lambda]$ else prices increase on an average, i.e. they are sub-martingale.

Proof. Appendix C □

For a range of beliefs, price on an average decreases over time. Charging a low price ensures a sale but does not improve future beliefs. High price improves next period beliefs and payoffs but does not ensure a sale. For a range of beliefs, the monopolist finds it profitable to take this bet and charge a high price.

To induce a consumer to purchase the product in the first period at a High price, the monopolist must compensate the consumer for his opportunity cost from waiting. Since, the opportunity cost is higher when the consumers have the option to wait, the price to induce a consumer to purchase the product is lower than when the cannot wait. Thus, in comparison to the case when a consumer can wait, the price sequence on an average increases over time for beliefs greater than $1-\lambda$.

Proposition 3. ***Trend in Expected Profits***
On an average, per period profit increases over time, i.e. the profit sequence is a sub-martingale.

Proof. See Appendix D □

We find that even though prices are a super martingale for initial low beliefs and sub martingale for higher beliefs, profits on an average increase over time. This is not only because prices are sub martingale for most of the beliefs but also because of the option to wait among consumers.

Comparison with Non Strategic Case

In this section we compare the pricing problem discussed in the previous sections to the one where consumers do not have the option to wait. First, we briefly describe the pricing strategy of the firm that faces consumers that have no option to wait, as a special case of Bose et al [4]. The value function of the firm in the second period in this case is:

$$V_2^{NS}(q_2) = Max\{0, 2f_l(q_2) - 1, q_2 - (1-\lambda)\}$$

The pricing strategy is the same as that of the firm that faces only one active consumer in the last period.

- For sufficiently low beliefs, the seller does not sell to any consumer.
- For intermediate beliefs, the seller sells the product such that only the High type consumer consumes it.
- For sufficiently high beliefs, the seller sells the product at the Low price such that all types of consumers consume it.

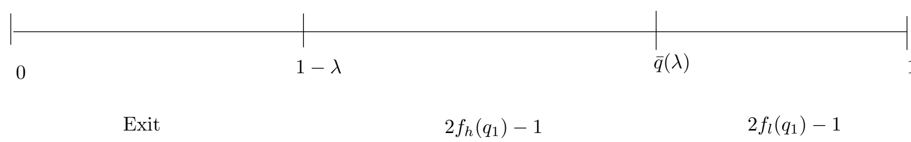

Figure 4: Last Period Pricing, Non strategic consumers

The value function of the firm that faces a consumer that has no option to wait in the first period is

$$V_1(q_1, a_1 = 1) = Max\{0, 2f_l(q_1) - 1 + \beta V_2(q_1, a_2 = 1), q_1 - (1-\lambda) + \beta EV_2(q_2, a_2 = 1)\}$$

Since, the firm can charge a price equal to the expected utility of either the High type or Low type period 1 consumer. The first period pricing strategy of the firm is a threshold policy. The pricing strategy is as following

- For sufficiently low beliefs ($q_1 \leq q'_{ns}$), the seller does not sell to any consumer.
- For intermediate beliefs, ($q_1 \in (q'_{ns}, q''_{ns})$), the seller sells the product such that only the High type consumer consumes it.
- For sufficiently high beliefs, ($q_1 \geq q''_{ns}$) the seller sells the product at the Low price such that there is always a consumption.

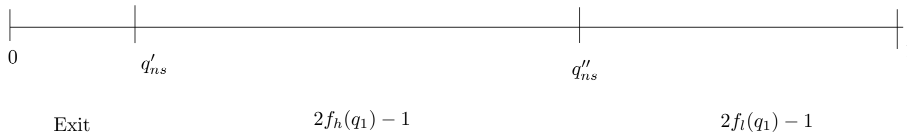

Figure 5: First Period Pricing, Consumers have no option to wait

q'_{ns} is defined such that

$$q'_{ns} - (1-\lambda) + \beta E V_2(q_2, a_2 = 1) = 0$$

and q''_{ns} is defined such that

$$q''_{ns} - (1-\lambda) + \beta E V_2(q_2, a_2 = 1) = 2f_l(q''_{ns}) - 1 + \beta V_2(q''_{ns}, a_2 = 1)$$

The next result finds that the range of beliefs where the firm encourages social learning is greater when consumers have the option to wait than when they do not.

Proposition 4. ***Greater Experimentation in Strategic Consumer Case***
The range of beliefs where the firm encourages social learning is greater in the strategic consumer case than in the non strategic consumer case.

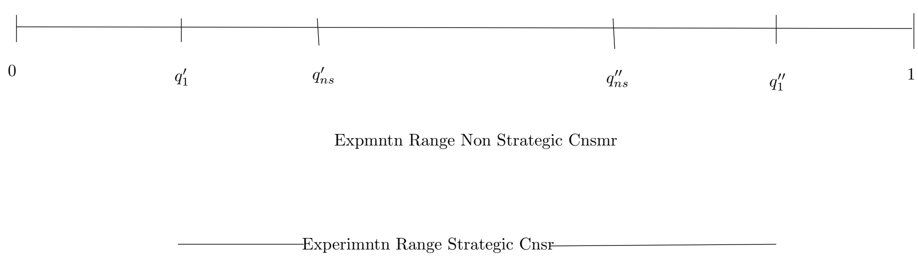

Figure 6: Greater Experimentation in Strategic Consumer Case

Proof. In the strategic consumer case, the firm introduces the product at the High price and thus encourages social learning for the range of beliefs $[q'_1, q''_1]$. When the firm faces non strategic consumers, the firm introduces the product at the High price and encourages social learning when beliefs is in the range, $[q'_{ns}, q''_{ns}]$.

We first prove that $q'_1 < q'_{ns}$. We know that q'_{ns} is such that

$$q'_{ns} - (1-\lambda) + \beta Pr(H)(q) V_2(f_h(q'_{ns}), a_2 = 1) + \beta Pr(L)(q) V_2(f_l(q'_{ns}), a_2 = 1) = 0$$

Also, q_1' is such that

$$q_1' - (1-\lambda) + \beta Pr(H)(q)V_2(f_h(q_1'), a_2 = 1) + \beta Pr(L)(q)V_2(f_l(q_1'), a_2 = 2) = 0$$

Since, $V_2(q, a_2 = 1) \leq V_2(q, a_2 = 2)$. Therefore, $q_1' \leq q_{ns}'$.
Next, we prove that $q_1'' > q_{ns}''$. We define q_{ns}'' is such that Consider the functions

$$\begin{aligned}\tilde{R_1}(q) &= [(1-\beta)(2f_h(q)-1) + \beta(2f_l(f_l(q)) - 1))]Pr(H)(q) \\ &+ \beta Pr(H)(q)V_2(f_h(q), a_2 = 1) + \beta Pr(L)(q)V_2(f_l(q), a_2 = 2)\end{aligned}$$

$$\tilde{R_2}(q) = q - (1-\lambda) + \beta Pr(H)(q)V_2(f_h(q), a_2 = 1) + \beta Pr(L)(q)V_2(f_l(q), a_2 = 1)$$

$\tilde{R_1}(q=1) \leq \tilde{R_2}(q=1)$ and $\tilde{R_1}(q = \bar{q}(\lambda)) \leq \tilde{R_2}(q = \bar{q}(\lambda))$. Since, $\tilde{R_1}(q)$ and $\tilde{R_2}(q)$ is a continuous and increasing function, for all values $q \in [\bar{q}(\lambda), 1]$, $\tilde{R_1}(q) \leq \tilde{R_2}(q)$. Thus, $q_1'' > q_{ns}''$. □

Encouraging learning or charging a price equivalent to the expected utility of the High type consumer comes at a cost of not ensuring a sale. However, it may improve future beliefs and profits. If the first period consumer can not wait and receives a low signal then the firm looses out on selling to this consumer by charging a High price. However, if the consumer can wait and receives a low signal then though the firm does not sell the product to the Low type consumer in the first period, it would do so in the next. Thus, the firm can hope to make a sale to the first period consumer in the strategic consumer case even if it charges a High price in the first period. Hence, the cost of improving next period beliefs is lower when the consumers can wait to purchase. Thus, the range of beliefs where learning is encouraged is higher in the strategic consumer case. Contrary to results in the literature, we find that the expected revenue is higher when consumers have the option to wait.

Proposition 5. ***Revenue Comparison***
For most range of initial beliefs, the expected revenue of the monopolist is no less in the case when consumers have the option to wait than when they do not.

Proof. See Appendix E □

The price of the product when the consumers have the option to wait is no greater than that when they do not. For initial common beliefs between, $[1-\lambda, f_h(q_2')]$ the last period expected pay off is the same in both the strategic and non strategic case (Either the product is not sold to any or is sold only to the High type consumer.)

For beliefs greater than $f_h(q_2')$, when consumers have the option to wait, the loss due to lower price charged in the first period is more than compensated by the gain due to higher expected number of sales. Thus, total expected revenue is higher when consumers have the option to wait because of the possibility of generating higher sales in the last period.

5 Conclusion

This paper analyzes optimal pricing strategy of a seller of a new durable good when both the seller and consumers learn about its unknown value. Previous prices and consumption decisions reveal information about the unknown value. A monopoly seller can influence the degree of learning by strategically choosing prices that influence social learning among strategic consumers. This is the first attempt in the literature to understand the optimal pricing strategy of the monopolist when it faces strategic consumers that consume a durable good after inspection.

We find that when the belief about the product quality is extreme, the monopoly seller either exits the market or introduces the product at a low price to everyone. However, for intermediate levels of the belief, the monopoly seller sells the product only to the high type consumers. We find that unlike the non strategic consumer case, the price sequence for a range of beliefs is a sub-martingale but otherwise prices on average decrease over time. We also find that the per period profits increases over time, i.e. the profits and consumption sequence are a sub-martingale.

Comparing the optimal pricing strategy of the firm when it faces non strategic consumers, we find that the range of beliefs where the firm encourages social learning in the first period is greater in the presence of strategic consumer case than in the non strategic consumer case. Also the monopolist gains revenue for most range of initial beliefs when it faces a strategic consumer in comparison to when consumers are non strategic.

Appendix

A Proof of Lemma 1

Proof. (i) When there is no delayed consumer in the second period, i.e. $d_2 = 0$, the monopolist faces only one fresh consumer and can charge one of two different prices,

$P^H(q_2)$ and $P^L(q_2)$. The expected profit from the two are as follows:

$$E^H(q_2) = q_2 - (1 - \lambda)$$

and

$$E^L(q_2) = 2f_L(q_2) - 1$$

The following diagram compares the expected profits from these two prices and hence the optimal pricing strategy.

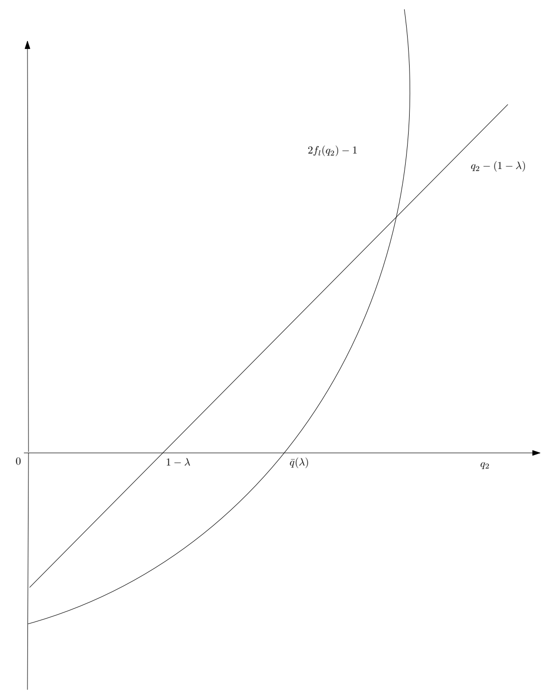

Figure 7: Comparison of Last Period Expected Profits, $d_2 = 0$

For beliefs less than $1 - \lambda$, the product is not introduced to any consumer. For beliefs between $1 - \lambda$ and $\bar{q}(\lambda)$ the product is sold at a price such that only the High type fresh consumers consume the product. For beliefs greater than $\bar{q}(\lambda)$ the product is sold at a price such that all types of fresh consumers consume the product.

(ii) When there is a delayed consumer in the second period, i.e. $d_2 = 1$, the monopolist can charge one of three different prices, $P^H(q_2)$, $P^M(q_2)$ and $P^L(q_2)$. The expected profit from the three pricing strategies is as follows:

$$E^H(q_2) = (q_2 - (1 - \lambda))$$

$$E^M(q_2) = [2q_2 - 1][1 + (\lambda q_2 + (1-\lambda)(1-q_2))]$$

and

$$E^L(q_2) = 2(2f_L(q_2) - 1)$$

The following diagram compares the expected profits from these three prices and hence the optimal pricing strategy.

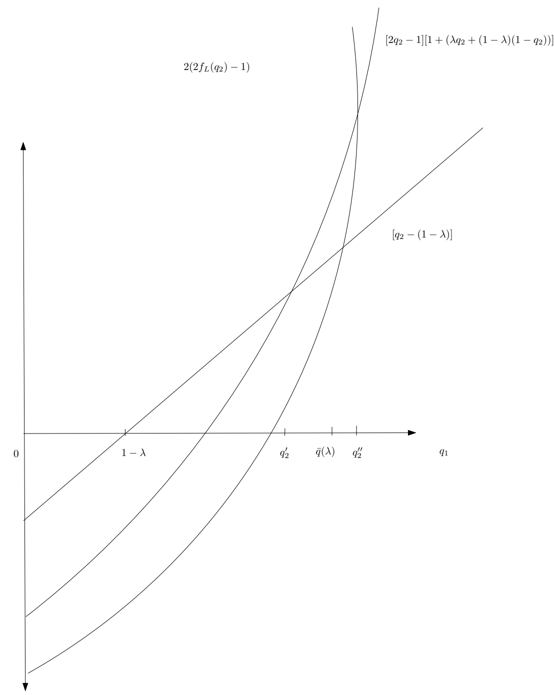

Figure 8: Comparison of Last Period Expected Profits, $d_2 \neq 0$

$E^H(q_2 = 0) = -1 + \lambda$, $E^M(q_2 = 0) = -2 + \lambda$, $E^L(q_2 = 0) = -2$
$E^H(q_2 = 1) = \lambda$, $E^M(q_2 = 1) = 1 + \lambda$, $E^L(q_2 = 1) = 2$

For beliefs less than $1 - \lambda$, $E^H(q_2), E^M(q_2), E^L(q_2)$ are all negative. Thus, for beliefs less than $1-\lambda$ the product is not introduced to any consumer. Since, $E^H(q_2 = 0) > E^M(q_2 = 0)$, $E^H(q_2 = 1) < E^M(q_2 = 1)$ and both $E^H(q_2), E^M(q_2)$ are continuous in q_2, there exists $q_2' = \frac{-1+\lambda+\sqrt{\lambda^2-\lambda+0.5}}{(2\lambda-1)}$ such that for beliefs less than q_2' the product is introduced only to the High type fresh consumer. For all beliefs greater than q_2' the firm prefers to introduce the product at a price equal to the expected utility of the delayed consumer than that of the High type fresh consumer.

Also, $E^M(q_2 = 0) > E^L(q_2 = 0)$, $E^M(q_2 = 1) < E^L(q_2 = 1)$ and both $E^M(q_2), E^L(q_2)$ are continuous in q_2, there exists q_2'' such that for beliefs less than q_2''

the product is introduced at a price such that both the High type fresh consumer and delayed consumer consumes the product. Also, for all beliefs less than q_2'' the product is introduced to sell to all.

We also note that $q_2' < \lambda < q_2''$ □

B Proof of Proposition 1

First we show that the only price that ensures that all types of fresh consumers consume the product is $2f_l(q_1) - 1$. Assume that the firm charges a price, $P_1(q_1)$ such that all types of fresh consumers consume the product. If there is a consumption at this price the next period belief would remain the same, i.e. $q_2 = q_1$.

Thus it must be true that both the Low and High type consumers consume the product at price $P_1(q_1)$. i.e.

and

$$2f_h(q_1) - 1 - P_1(q_1) \geq \beta Max\{0, 2f_h(q_1) - 1 - P_2(q_1, d_2 = 1)\}$$

Given that $P_1(q_1)$ is such that next period beliefs remain the same, price in the second period, $P_2(q_1, d_2 = 1)$ can be either $2f_h(q_1) - 1$, $2q_1 - 1$, $2f_l(q_1) - 1$. Given this the only price which satisfies the two equations above is $2f_l(q_1) - 1$.

Next, we show that the price which sells only to the High type consumer, $P_1^H(q_1)$ is

$$P_1^H(q_1) = \begin{cases} 2f_h(q_1) - 1; & q_1 < 1 - \lambda \\ (1-\beta)(2f_h(q_1) - 1) & q_1 \in [1-\lambda, 0.5) \\ (1-\beta)(2f_h(q_1) - 1) + \beta(2q_1 - 1) & q_1 \in [0.5, f_h(q_2')) \\ (1-\beta)(2f_h(q_1) - 1) + \beta(2f_l(q_1) - 1) & q_1 \in [f_h(q_2'), f_h(q_2'')) \\ (1-\beta)(2f_h(q_1) - 1) + \beta(2f_l(f_l(q_1)) - 1) & q_1 \in [f_h(q_2''), 1] \end{cases}$$

The price $P_1^H(q_1)$ must be such that only the High type consumer consumes the product. In this case if there is no consumption in the first period then the next period belief would be $q_2 = f_l(q_1)$.

$$2f_h(q_1) - 1 - P_1(q_1) \geq \beta Max\{0, 2f_h(q_1) - 1 - P_2(f_l(q_1), d_2 \neq 0)\}$$
$$2f_l(q_1) - 1 - P_1(q_1) < \beta Max\{0, f_l(q_1) - 1 - P_2(f_l(q_1), d_2 \neq 0)\}$$

Now, $P_2(f_l(q_1), d_2 \neq 0)$ takes different values depending on q_1.

$$P_2(f_l(q_1), d_2 \neq 0) = \begin{cases} 0; & q_1 < 0.5 \\ (2f_h(f_l(q_1)) - 1) & q_1 \in [0.5, f_h(q_2')) \\ (2f_l(q_1) - 1) & q_1 \in [f_h(q_2'), f_h(q_2'')) \\ (2f_l(f_l(q_1)) - 1) & q_1 \in [f_h(q_2''), 1] \end{cases}$$

Thus, the only price that satisfies these two conditions is

$$P_1^H(q_1) = \begin{cases} 2f_h(q_1) - 1 & q_1 < 1 - \lambda \\ (1-\beta)(2f_h(q_1) - 1) & q_1 \in [1-\lambda, 0.5) \\ (1-\beta)(2f_h(q_1) - 1) + \beta(2q_1 - 1) & q_1 \in [0.5, f_h(q_2')) \\ (1-\beta)(2f_h(q_1) - 1) + \beta(2f_l(q_1) - 1) & q_1 \in [f_h(q_2'), f_h(q_2'')) \\ (1-\beta)(2f_h(q_1) - 1) + \beta(2f_l(f_l(q_1)) - 1) & q_1 \in [f_h(q_2''), 1] \end{cases}$$

The value function faced by the monopolist in the first period is

$$\begin{aligned} V_1(q_1) \;\; = \;\; & Max\{0, 2f_l(q_1) - 1 + \beta V_2(q_1, d_2 = 0), \\ & P_1^H(q_1)Pr(H)(q) + \beta Pr(H)(q)V_2(f_h(q_1), d_2 = 0) + Pr(L)(q)\beta V_2(f_l(q_1), d_2 \neq 0)\} \end{aligned}$$

where, $Pr(H)(q) = (\lambda q + (1-\lambda)(1-q))$ and $Pr(L)(q) = (1-\lambda)q + \lambda(1-q)$

$$\begin{aligned} & \beta Pr(H)(q)V_2(f_h(q_1), d_2 = 0) + \beta Pr(L)V_2(f_l(q_1), d_2 \neq 0) \\ \geq \;\; & \beta Pr(H)(q)V_2(f_h(q_1), d_2 = 0) + \beta Pr(L)V_2(f_l(q_1), d_2 = 0)\} \\ & (\because V_2(q_2, d_2 \neq 0) \geq V_2(q_2, d_2 = 0)) \\ \geq \;\; & V_2(q_1, d_2 = 0) \end{aligned}$$

For beliefs $q_1 \in [\lambda, f_h(q_2')]$

$$Pr(H)(q)((1-\beta)(2f_h(q_1) - 1) + \beta(2q_1 - 1)) > 2f_l(q_1) - 1$$

And for beliefs $q_1 \in [f_h(q_2'), f_h(q_2'')]$

$$Pr(H)(q)((1-\beta)(2f_h(q_1) - 1) + \beta(2f_l(q_1) - 1)) > 2f_l(q_1) - 1$$

Therefore, the value function of the monopolist can be written as

$$\begin{aligned} V_1(q_1) \;\; = \;\; & Max\{0, P_1^H(q_1)Pr(H)(q) + \beta Pr(H)(q)V_2(f_h(q_1), d_2 = 0) \\ + \;\; & \beta Pr(L)(q)V_2(f_l(q_1), d_2 \neq 0) \forall q \leq f_h(q_2'')\} \\ = \;\; & Max\{0, 2f_l(q_1) - 1 + \beta V_2(q_1, d_2 = 0), \\ & P_1^H(q_1)Pr(H)(q) + \beta Pr(H)(q)V_2(f_h(q_1), d_2 = 0) \\ + \;\; & \beta Pr(L)(q)V_2(f_l(q_1), d_2 \neq 0) \forall q \geq f_h(q_2'')\} \end{aligned}$$

Consider the function, $L_1(q_1) = P_1^H(q_1)Pr(H)(q) + \beta Pr(H)(q)V_2(f_h(q_1), d_2 = 0) + \beta Pr(L)(q)V_2(f_l(q_1), d_2 \neq 0)$. $L_1(q_1 = 0) = -(1-\lambda)$ and $L_1(q_1 = 1-\lambda) \geq 0$. Since, $L_1(q_1)$ is an increasing continuous function there exists a threshold $q_1' < 1-\lambda < f_h(q_2'')$ such that for all beliefs less than q_1' the firm exits the market and for all beliefs greater than q_1' the firm sells the product at the High price, $P_1^H(q_1)$. For beliefs greater than $q_1 \geq q_1' \geq f_h(q_2'')$, the monopolist chooses between introducing the product at the High price or selling the product to all types of fresh consumers. Consider the function $L_2(q_1) = 2f_l(q_1) - 1 + \beta V_2(q_1, d_2 = 0)$. $L_2(q_1 = 1-\lambda) \leq 0$ and $L_2(q_1 = \bar{q}(\lambda)) \geq L_1(q_1 = \bar{q}(\lambda))$. Thus, there exists $q_1'' \in [f_h(q_2''), \bar{q}(\lambda)]$ such that $\forall q_1 \leq q_1''$ the firm prefers to introduce the product at the High price than to sell it to all consumers. However, for beliefs greater than q_1' the firm sells the product at the Low price to all types of fresh consumers. Thus, the first period pricing strategy of the firm is as follows: For beliefs higher than q_1' the price of the product is as following:

$$P_1(q_1) = \begin{cases} 0; & q_1 \leq q_1' \\ 2f_h(q_1) - 1; & q_1 \in [q_1', 1-\lambda] \\ (1-\beta)(2f_h(q_1) - 1) & q_1 \in [1-\lambda, 0.5) \\ (1-\beta)(2f_h(q_1) - 1) + \beta(2q_1 - 1) & q_1 \in [0.5, f_h(q_2')) \\ (1-\beta)(2f_h(q_1) - 1) + \beta(2f_l(q_1) - 1) & q_1 \in [f_h(q_2'), f_h(q_2'')) \\ (1-\beta)(2f_h(q_1) - 1) + \beta(2f_l(f_l(q_1)) - 1) & q_1 \in [f_h(q_2''), q_1''] \\ 2f_l(q_1) - 1 & q_1 \geq q_1'' \end{cases}$$

C Proof of Proposition 2

Proof. The belief and price changes in the next period only when the first period beliefs are in the range $q_1 \in [q_1', q_1'']$.

- For beliefs, $q_1 \in [q_1', 1-\lambda]$ the first period price is $P_1^H(1)(q_1) = 2f_h(q_1) - 1$. Since, $f_l(1-\lambda) < 1-\lambda$ if the first period consumer receives a low signal then the next period beliefs, $q_2 = f_l(q_1)$ would be such that the product would not be sold to any consumer. However, if the first period fresh consumer receives a high signal and the beliefs improve to $q_2 = f_h(q_1)$ then the next period beliefs would be such that the product is introduced at the High price, $2f_h(q_2) - 1$ or $2f_h(f_h(q_1)) - 1$. Thus, the expected price in the second period would be

$$E[P_2(1)(q_1)|P_1] = [\lambda q_1 + (1-\lambda)(1-q_1)][2f_h(f_h(q_1)) - 1]$$

Since, $f_h(q_1)$ is a concave function in q_1, $E[P_2(1)|P_1] < P_1^H(1)$.

- For beliefs, $q_1 \in [1-\lambda, 0.5]$ the first period price is $P_1^H(2)(q_1) = (1-\beta)(2f_h(q_1)-1)$. Since, $f_l(0.5) = 1-\lambda$ if the first period consumer receives a low signal then the next period beliefs, $q_2 = f_l(q_1)$ would be such that the product would not be sold to any consumer. However, if the first period fresh consumer receives a high signal and the beliefs improve to $q_2 = f_h(q_1)$ then the next period beliefs would be such that the product is introduced at the High price, $2f_h(q_2) - 1$ or $2f_h(f_h(q_1)) - 1$. Thus, the expected price in the second period would be

$$E[P_2|P_1^H(2)(q_1)] = [\lambda q_1 + (1-\lambda)(1-q_1)][2f_h(f_h(q_1)) - 1]$$

Consider the functions $P_1^H(2)(q_1)$ and $E[P_2(2)(q_1)|P_1]$. $E[P_2(2)(q_1 = 0)|P_1] = -1+\lambda$ and $P_1^H(2)(q_1 = 0) = -1+\beta$. $E[P_2(2)(q_1 = 1)|P_1] = \lambda$ and $P_1^H(2)(q_1 = 1) = 1-\beta$. If $\beta < \lambda$ then $E[P_2(2)(q_1)|P_1] > P_1^H(2)(q_1)\forall q_1$. However, if $\beta > \lambda$ then $E[P_2(2)(q_1 = 1-\lambda)|P_1] > P_1^H(2)(q_1 = 1-\lambda)$ and $E[P_2(2)(q_1 = 0)|P_1] < P_1^H(2)(q_1 = 0)$. Therefore, $\forall q_1 \geq 1-\lambda E[P_2(2)(q_1)|P_1] > P_1^H(2)(q_1)$. Therefore, $E[P_2|P_1^H(2)(q_1)] \geq P_1^H(2)(q_1)$.

- For beliefs, $q_1 \in [0.5, f_h(q_2')]$ the first period price is $P_1^H(3)(q_1) = (1-\beta)(2f_h(q_1)-1) + \beta(2q_1 - 1)$. Since, $f_l(0.5) = 1-\lambda$ and $f_l(q_1)$ is an increasing function, if the first period consumer receives a low signal then the next period beliefs, $q_2 = f_l(q_1)$ would be such that the product would be sold to a consumer at the High price, i.e. $2f_h(q_2) - 1$ or $2f_h(f_l(q_1)) - 1$. However, if the first period fresh consumer receives a high signal and the beliefs improve to $q_2 = f_h(q_1)$ then the next period beliefs would be such that the product is introduced at the High price, $2f_h(q_2) - 1$ or $2f_h(f_h(q_1)) - 1$. Thus, the expected price in the second period would be

$$\begin{aligned} E[P_2(3)(q_1)|P_1^H(3)] &= [\lambda q_1 + (1-\lambda)(1-q_1)][2f_h(f_h(q_1)) - 1] \\ &+ [(1-\lambda)q_1 + \lambda(1-q_1)][2f_h(f_l(q_1)) - 1] \end{aligned}$$

Consider the functions $P_1^H(3)(q_1)$ and $E[P_2(3)(q_1)|P_1^H(3)]$. $E[P_2(q_1 = 0)|P_1^H(3)] = -1$ and $P_1^H(3)(q_1 = 0) = -1$. $E[P_2(3)(q_1 = 1)|P_1^H(3)] = 1$ and $P_1^H(3)(q_1 = 1) = 1$. Both $E[P_2(3)(q_1)|P_1^H(3)]$, $P_1^H(3)(q_1)$ are increasing and concave functions of q_1. At $E[P_2(3)(q_1 = 0.5)|P_1^H(3)] = (2\lambda - 1)(1-\beta)$ and $P_1^H(3)(q_1 = 0.5) = \frac{2\lambda-1}{2(\lambda^2+(1-\lambda)^2)}$.

- For beliefs, $q_1 \in [f_h(q_2'), f_h(q_2'')]$ the first period price is $P_1^H(4)(q_1) = (1-\beta)(2f_h(q_1) - 1) + \beta(2f_l(q_1) - 1)$. Since, $f_l(f_h(q_2')) = q_2'$ and $f_l(f_h(q_2'')) = q_2''$ if the first period consumer receives a low signal then the next period beliefs, $q_2 = f_l(q_1)$ would be such that the product would be sold at the price of the expected utility of the delayed consumer, $2q_2 - 1 = 2f_l(q_1) - 1$. However, if the first period fresh consumer receives a high signal and the beliefs improve to $q_2 = f_h(q_1)$ then the next period beliefs would be such that the product is introduced at the High price, $2f_h(q_2) - 1$ or $2f_h(f_h(q_1)) - 1$.

$$\begin{aligned} E[P_2(4)(q_1)|P_1] &= [\lambda q_1 + (1-\lambda)(1-q_1)][2f_h(f_h(q_1)) - 1] \\ &+ [(1-\lambda)q_1 + \lambda(1-q_1)][2f_l(q_1) - 1] \end{aligned}$$

Consider the functions $P_1^H(4)(q_1)$ and $E[P_2(4)(q_1)|P_1]$. $E[P_2(4)(q_1 = 0)|P_1^H(4)] = -1$ and $P_1^H(4)(q_1 = 0) = -1$. $E[P_2(4)(q_1 = 1)|P_1^H(4)] = 1$ and $P_1^H(4)(q_1 = 1) = 1$. Both $E[P_2(4)(q_1)|P_1^H(4)]$, $P_1^H(4)(q_1)$ for $\beta \geq 0.5$ are increasing and convex functions of q_1. For $\beta \geq 2/3$, $E[P_2(4)(q_1 = \frac{\lambda^2}{\lambda^2+(1-\lambda)^2})|P_1^H(4)] \geq P_1^H(4)(q_1 = \frac{\lambda^2}{\lambda^2+(1-\lambda)^2})$. Therefore, $E[P_2(4)(q_1)|P_1^H] \geq P_1^H(4)(q_1)\forall q_1$.

- For beliefs, $q_1 \in [f_h(q_2''), q_1']$ the first period price is $P_1^H(5)(q_1) = (1-\beta)(2f_h(q_1)-1) + \beta(2f_l(f_l(q_1)) - 1)$. Since, $f_l(f_h(q_2'')) = q_2''$ if the first period consumer receives a low signal then the next period beliefs, $q_2 = f_l(q_1)$ would be such that the product would be sold to all types of consumers at the price, $2f_l(q_2) - 1$. However, if the first period fresh consumer receives a high signal and the beliefs improve to $q_2 = f_h(q_1)$ then the next period beliefs would be such that the product is introduced at the Low price, $2f_l(q_2) - 1$ or $2f_l(f_h(q_1)) - 1$ since, $q_2'' \geq \bar{q}(\lambda)$.

$$\begin{aligned} E[P_2(5)(q_1)|P_1^H(5)] &= [\lambda q_1 + (1-\lambda)(1-q_1)][2f_l(f_h(q_1)) - 1] \\ &+ [(1-\lambda)q_1 + (\lambda)(1-q_1)][2f_l(f_l(q_1)) - 1] \end{aligned}$$

Consider the functions $P_1^H(5)(q_1)$ and $E[P_2(5)(q_1)|P_1^H(5)]$. $E[P_2(5)(q_1 = 0)|P_1^H(5)] = -1$ and $P_1^H(5)(q_1 = 0) = -1$. $E[P_2(q_1 = 1)|P_1^H(5)] = 1$ and $P_1^H(5)(q_1 = 1) = 1$. Both $E[P_2(q_1)|P_1^H(5)]$, $P_1^H(5)(q_1)$ for $\beta \geq 0.5$ are increasing and convex functions of q_1. For $\beta \geq 2/3$, $E[P_2(q_1 = \frac{\lambda^2}{\lambda^2+(1-\lambda)^2})|P_1^H(5)] \geq P_1^H(5)(q_1 = \frac{\lambda^2}{\lambda^2+(1-\lambda)^2})$. Therefore, $E[P_2(5)(q_1)|P_1^H] \geq P_1^H(5)(q_1)\forall q_1$.

$\square$

D Proof of Proposition 3

The belief and profits changes in the next period only when the first period beliefs are in the range $q_1 \in [q_1', q_1'']$.

- For beliefs, $q_1 \in [q_1', 1-\lambda]$ the first period expected profit is $\Pi_1(1)(q_1) = q_1 - (1-\lambda)$. If the first period consumer receives a Low signal then the next period beliefs, $q_2 = f_l(q_1)$ would be such that the product would not be sold to any consumer. However, if the first period fresh consumer receives a good signal next period beliefs improve to $q_2 = f_h(q_1)$. Next period beliefs would be such that the product is introduced at the High price, $2f_h(q_2)-1$ or $2f_h(f_h(q_1))-1$. Thus, the expected profit in the second period would be

  $$E[\Pi_2(1)(q_1)|\Pi_1(1)] = [\lambda q_1 + (1-\lambda)(1-q_1)][f_h(q_1) - (1-\lambda)]$$

  Since, $q_1 \leq 0.5$, $E[\Pi_2(1)(q_1)|\Pi_1(1)] > \Pi_1(1)(q_1)$.

- For beliefs, $q_1 \in [1-\lambda, 0.5]$ first period profit is $\Pi_1(2)(q_1) = (1-\beta)(q_1-(1-\lambda))$. Since, $f_l(0.5) = 1-\lambda$ if the first period consumer receives a Low signal then the next period beliefs would be, $q_2 = f_l(q_1)$ and the product would not be sold to any consumer. However, if the first period fresh consumer receives a good signal and the beliefs improve to $q_2 = f_h(q_1)$ then next period beliefs would be such that the product is introduced at the High price, $2f_h(q_2)-1$ or $2f_h(f_h(q_1))-1$. Thus, the expected profit in the second period would be

  $$E[\Pi_2(2)(q_1)|\Pi(2)_1] = [\lambda q_1 + (1-\lambda)(1-q_1)][f_h(q_1) - (1-\lambda)]$$

  Consider the functions $\Pi_1(2)(q_1)$ and $E[\Pi_2(2)(q_1)|\Pi_1]$. $E[\Pi_2(2)(q_1=0)|\Pi_1(2)] = -(1-\lambda)^2$ and $\Pi_1(2)(q_1=0) = -(1-\beta)(1-\lambda)$. $E[\Pi_2(2)(q_1=1)|\Pi_1(2)] = \lambda^2$ and $\Pi_1(2)(q_1=1) = \lambda^2$. For $\beta \geq 2/3$, $E[\Pi_2(2)(q_1=1)|\Pi_1(2)] \geq \Pi_1(2)(q_1=1)$. Also, $E[\Pi_2(2)(q_1=1-\lambda)|\Pi_1(2)] \geq \Pi_1(2)(q_1=1-\lambda)$. Since, both $E[\Pi_2(2)(q_1)|\Pi_1(2)]$ and $\Pi_1(2)(q_1)$ are increasing in q_1, $\forall q_1 \geq 1-\lambda$, $E[\Pi_2(2)(q_1)|\Pi_1(2)] \geq \Pi_1(2)(q_1)$.

- For beliefs, $q_1 \in [0.5, f_h(q_2')]$ the first period profit is $\Pi_1(3)(q_1) = (1-\beta)(q_1-(1-\lambda))+\beta(\lambda q_1+(1-\lambda)(1-q_1))(2q_1-1)$. Since, $f_l(0.5) = 1-\lambda$ and $f_l(q_1)$ is an increasing function, if the first period consumer receives a Low signal then the next period beliefs, $q_2 = f_l(q_1)$ would be such that the product would be sold to a consumer at the High price, i.e. $2f_h(q_2)-1$ or $2f_h(f_l(q_1))-1$. However, if the first period fresh consumer receives a good signal and the beliefs improve to $q_2 = f_h(q_1)$ then the next period beliefs would be such that the product is introduced at the High price, $2f_h(q_2)-1$ or $2f_h(f_h(q_1))-1$. Thus, the expected profit in the second period would be

  $$\begin{aligned} E[\Pi_2(3)(q_1)|\Pi_1(3)] &= [\lambda q_1 + (1-\lambda)(1-q_1)][f_h(q_1) - (1-\lambda)] \\ &+ [(1-\lambda)q_1 + (\lambda)(1-q_1)][2f_h(f_l(q_1)) - 1] \\ &\quad (\lambda f_l(q_1) + (1-\lambda)(1-f_l(q_1))) \end{aligned}$$

Consider the functions $\Pi_1(3)(q_1)$ and $E[\Pi_2(3)(q_1)|\Pi_1(3)]$.

$E[\Pi_2(3)(q_1 = 0)|\Pi_1(3)] = -1 + \lambda$ and $\Pi_1(3)(q_1 = 0) = -1 + \lambda$. $E[\Pi_2(3)(q_1 = 1)|\Pi_1(3)] = \lambda$ and $\Pi_1(3)(q_1 = 1) = \lambda$. Both $E[\Pi_2(3)(q_1)|\Pi_1(3)]$, $\Pi_1(3)(q_1)$ are increasing and convex functions of q_1. At $q_1 = 1 - \lambda$, $E[\Pi_2(3)(q_1 = 1 - \lambda)|\Pi_1(3)] \geq \Pi_1(3)(q_1 = 1 - \lambda)$. Therefore, $\forall q_1 E[\Pi_2(3)(q_1)|\Pi_1(3)] \geq \Pi_1(3)(q_1)$.

- For beliefs, $q_1 \in [f_h(q_2'), f_h(q_2'')]$ the first period profit is $\Pi_1(4)(q_1) = (1 - \beta)(q_1 - (1-\lambda)) + \beta(2f_l(q_1) - 1)(\lambda q_1 + (1-\lambda)(1-q_1)$. Since, $f_l(f_h(q_2')) = q_2'$ and $f_l(f_h(q_2'')) = q_2''$ if the first period consumer receives a low signal then the next period beliefs, $q_2 = f_l(q_1)$ would be such that the product would be sold at the price of the expected utility of the delayed consumer, $2q_2 - 1 = 2f_l(q_1) - 1$. However, if the first period fresh consumer receives a high signal and the beliefs improve to $q_2 = f_h(q_1)$ then the next period beliefs would be such that the product is introduced at the High price, $2f_h(q_2) - 1$ or $2f_h(f_h(q_1)) - 1$. Thus, the next period expected profits would be

$$\begin{aligned} E[\Pi_2(4)(q_1)|\Pi_1] &= [\lambda q_1 + (1-\lambda)(1-q_1)][f_h(q_1) - (1-\lambda)] \\ &+ [(1-\lambda)q_1 + (\lambda)(1-q_1)][2f_l(q_1) - 1] \\ &\quad (1 + \lambda f_l(q_1) + (1-\lambda)(1 - f_l(q_1)) \\ &= 2q_1 - 1 \end{aligned}$$

  Consider the functions $\Pi(4)(q_1)$ and $E[\Pi_2(4)(q_1)|\Pi_1]$. $E[\Pi_2(4)(q_1 = 0)|\Pi_1] = -1$ and $\Pi_1(4)(q_1 = 0) = -1+\lambda$. $E[\Pi_2(4)(q_1)|\Pi_1(4)] = 1$ and $\Pi_1(4)(q_1 = 1) = \lambda$.

  Both $E[\Pi_2(4)(q_1)|\Pi_1]$, $\Pi_1(q_1)$ are increasing and convex functions of q_1. At $q_1 = \lambda$, $E[\Pi_2(4)(q_1 = \lambda)|\Pi_1(4)] \geq \Pi_1(4)(q_1 = \lambda)$ and for $q_1 = 0.5$, $E[\Pi_2(4)(q_1 = 0.5)|\Pi_1(4)] \leq \Pi_1(4)(q_1 = 0.5)$. Since, $\lambda \leq f_h(q_2')$, $E[\Pi_2(4)(q_1)|\Pi_1(4)] \geq \Pi_1(4)(q_1)$ $\forall q_1 \in [f_h(q_2'), f_h(q_2'')]$. Therefore, $E[\Pi_2(4)(q_1)|\Pi_1(4)] \geq \Pi_1(4)(q_1)$.

- For beliefs, $q_1 \in [f_h(q_2''), q_1']$ the first period profits is $\Pi_1(5)(q_1) = (1-\beta)(q_1 - (1-\lambda)) + \beta(\lambda q_1 + (1-\lambda)(1-q_1))(2f_l(f_l(q_1)) - 1)$. Since, $f_l(f_h(q_2'')) = q_2''$ if the first period consumer receives a low signal then the next period beliefs, $q_2 = f_l(q_1)$ would be such that the product would be sold to all types of consumers at the price, $2f_l(q_2) - 1$. However, if the first period fresh consumer receives a high signal and the beliefs improve to $q_2 = f_h(q_1)$ then the next period beliefs would be such that the product is introduced at the Low price, $2f_l(q_2) - 1$ or $2f_l(f_h(q_1)) - 1$ since, $q_2'' \geq \bar{q}(\lambda)$. Thus, the expected profits in the next period would be

$$\begin{aligned} E[\Pi_2(5)(q_1)|\Pi_1] &= [\lambda q_1 + (1-\lambda)(1-q_1)][2f_l(f_l(q_1)) - 1] \\ &+ [(1-\lambda)q_1 + (\lambda)(1-q_1)][2f_l(f_h(q_1)) - 1] \end{aligned}$$

Consider the functions $\Pi_1(5)(q_1)$ and $E[\Pi_2(5)(q_1)|\Pi_1(5)]$.

$E[\Pi_2(5)(q_1 = 0)|\Pi_1(5)] = -1$ and $P_1^H(5)(q_1 = 0) = -1+\lambda$. $E[\Pi_2(5)(q_1)|\Pi_1(5)] = 1$ and $\Pi_1(5)(q_1 = 1) = \lambda$. Both $E[\Pi_2(5)(q_1)|\Pi_1(5)]$, $\Pi_1(5)(q_1)$ are increasing and convex functions of q_1. For $q_1 \geq f_h(q_2'')$ $E[\Pi_2(5)(q_1)|\Pi_1(5)] \geq \Pi_1(5)(q_1)$. Therefore, $E[\Pi_2(5)|\Pi_1(5)] \geq \Pi_1(5)(q_1)$.

E Proof of Lemma 4

From Lemma 3 we know that $q_1'' < q_{ns}'$ and $q_1'' > q_{ns}''$.

- For beliefs less than q_1' the firm when it faces strategic and non strategic consumers would not introduce the product to any type of consumer. Hence, the expected profits and the difference in the profit is zero.

- For beliefs $q_1 \in [q_1', q_{ns}']$, the firm introduces the product to high type consumers at price $2f_h(q_1)-1$ when it faces strategic consumers and generates an expected profit of $q_1 - (1-\lambda) + \beta(\lambda q_1 + (1-\lambda)(1-q_1))V_2(f_h(q_1), d_2 = 0)$. When the firm faces non strategic consumers the product is not introduced to any type of consumer for this range of beliefs. Thus, the expected profit in the non strategic consumer case is lower than that in the strategic consumer case for this range of beliefs.

- For beliefs $q_1 \in [q_{ns}', 1-\lambda]$, the firm introduces the product to high type consumer at price $2f_h(q_1) - 1$ both when it faces strategic and non strategic consumers. Since, in this range $V_2(f_l(q_1), d_2 = 0)$ and $V_2(f_l(q_1), d_2 = 1)$ are the same the expected revenue in both these scenario is the same for this range of beliefs.

- For beliefs $q_1 \in [1-\lambda, 0.5]$, the firm introduces the product to high type consumer at price $\beta[2f_h(q_1)-1]$ when it faces strategic consumers and generates an expected profit of $\beta[q_1-(1-\lambda)]+\beta(\lambda q_1+(1-\lambda)(1-q_1))V_2(f_h(q_1), d_2 = 0)$ (as $V_2(f_l(q_1), d_2 = 1)$ is zero for this range of beliefs). Similarly, the firm introduces the product to high type consumer at price $[2f_h(q_1) - 1]$ when it faces non strategic consumers and generates an expected profit of $\beta[q_1 - (1-\lambda)] + \beta(\lambda q_1 + (1-\lambda)(1-q_1))V_2(f_h(q_1), d_2 = 0)$. Thus, the expected profits in the non strategic consumer case is higher than that in the strategic consumer case for this range of beliefs.

- For beliefs $q_1 \in [0.5, f_h(q_2')]$, the firm introduces the product to high type consumer at price $2f_h(q_1) - 1$ when the consumers are non strategic. The expected profit from this is $q_1 - (1-\lambda) + \beta(\lambda q_1 + (1-\lambda)(1-q_1))V_2(f_h(q_1), d_2 = 0) + \beta(\lambda(1-q_1) + (1-\lambda)q_1)V_2(f_l(q_1), d_2 = 0)$. However, in the strategic consumer case the product is introduced to the high type consumers at the price $(1-\beta)(2f_h(q_1) - 1) + \beta(2q_1 - 1)$. The expected profit in this case would be $(1-\beta)(q_1 - (1-\lambda)) + \beta(2q_1 - 1)(\lambda q_1 + (1-\lambda)(1-q_1)) + \beta(\lambda q_1 + (1-\lambda)(1-q_1))V_2(f_h(q_1), d_2 = 0) + \beta(\lambda(1-q_1) + \lambda(1-q_1))V_2(f_l(q_1), d_2 \neq 0)$. For this range of beliefs, $V_2(f_l(q_1), d_2 = 0) = V_2(f_l(q_1), d_2 \neq 0)$. Therefore, the expected profits from the non strategic consumer case is higher than that in the strategic consumer case.

- For beliefs $q_1 \in [f_h(q_2'), \bar{q}(\lambda)]$, the firm introduces the product to high type consumer at price $2f_h(q_1) - 1$ when the consumers are non strategic. The expected profit from this is

$$\begin{aligned}\Pi_{NS}(q_1) &= q_1 - (1-\lambda) + \beta(\lambda q_1 + (1-\lambda)(1-q_1))V_2(f_h(q_1), d_2 = 0) \\ &+ \beta(\lambda(1-q_1) + (1-\lambda)q_1)(2q_1 - 1)(\lambda f_l(q_1) + (1-\lambda)(1-f_l(q_1)))\end{aligned}$$

  However, in the strategic consumer case the product is introduced to the high type consumers at the price $(1-\beta)(2f_h(q_1) - 1) + \beta(2f_l(q_1) - 1)$. The expected profit in this case would be

$$\begin{aligned}\Pi_S(q_1) &= (1-\beta)(q_1 - (1-\lambda)) + \beta(2f_l(q_1) - 1)(\lambda q_1 + (1-\lambda)(1-q_1)) \\ &+ \beta(\lambda q_1 + (1-\lambda)(1-q_1))V_2(f_h(q_1), d_2 = 0) \\ &+ \beta(\lambda(1-q_1) + \lambda(1-q_1))(2f_l(q_1) - 1)(1 + \lambda f_l(q_1) + (1-\lambda)(1-f_l(q_1)))\end{aligned}$$

  For beliefs $q_1 \in [f_h(q_2'), \bar{q}(\lambda)]$, $\Pi_S(q_1) > \Pi_{NS}(q_1)$.

- For beliefs $q_1 \in [\bar{q}(\lambda), f_h(q_2'')]$, the firm introduces the product to high type consumer at price $2f_h(q_1) - 1$ when the consumers are non strategic. The expected profit from this is

$$\begin{aligned}\Pi_{NS}(q_1) &= q_1 - (1-\lambda) + \beta(\lambda q_1 + (1-\lambda)(1-q_1))V_2(f_h(q_1), d_2 = 0) \\ &+ \beta(\lambda(1-q_1) + (1-\lambda)q_1)(2q_1 - 1)(\lambda f_l(q_1) + (1-\lambda)(1-f_{ll}(q_1)))\end{aligned}$$

  However, in the strategic consumer case the product is introduced to the high type consumers at the price $(1-\beta)(2f_h(q_1) - 1) + \beta(2f_l(q_1) - 1)$. The expected profit in this case would be

$$\begin{aligned}\Pi_S(q_1) &= (1-\beta)(q_1-(1-\lambda))+\beta(2f_l(q_1)-1)(\lambda q_1+(1-\lambda)(1-q_1))\\ &+ \beta(\lambda q_1+(1-\lambda)(1-q_1))V_2(f_h(q_1),d_2=0)\\ &+ \beta(\lambda(1-q_1)+\lambda(1-q_1))(2f_l(q_1)-1)(1+\lambda f_l(q_1)+(1-\lambda)(1-f_l(q_1)))\end{aligned}$$

For beliefs $q_1 \in [\bar{q}(\lambda), f_h(q_2'')]$, $\Pi_S(q_1) > \Pi_{NS}(q_1)$.

- For beliefs $q_1 \in [f_h(q_2''), q_1'']$, the firm introduces the product to high type consumer at price $2f_h(q_1)-1$ when the consumers are non strategic. The expected profit from this is

$$\begin{aligned}\Pi_{NS}(q_1) &= q_1-(1-\lambda)+\beta(\lambda q_1+(1-\lambda)(1-q_1))V_2(f_h(q_1),d_2=0)\\ &+ \beta(\lambda(1-q_1)+(1-\lambda)q_1)2(2f_{ll}(q_1)-1)\end{aligned}$$

However, in the strategic consumer case the product is introduced to the high type consumers at the price $(1-\beta)(2f_h(q_1)-1)+\beta(2f_{ll}(q_1)-1)$. The expected profit in this case would be

$$\begin{aligned}\Pi_S(q_1) &= (1-\beta)(q_1-(1-\lambda))+\beta(2f_{ll}(q_1)-1)(\lambda q_1+(1-\lambda)(1-q_1))\\ &+ \beta(\lambda q_1+(1-\lambda)(1-q_1))V_2(f_h(q_1),d_2=0) \qquad (1)\\ &+ \beta(\lambda(1-q_1)+\lambda(1-q_1))2(2f_{ll}(q_1)-1)\end{aligned}$$

For beliefs $q_1 \in [f_h(q_2''), q_1'']$, $\Pi_S(q_1) > \Pi_{NS}(q_1)$.

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