Inconsistency in Response when Questions are Repeated with the Number of Options Changing

by

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Abstract

Discarding haphazard and insincere respondents can improve the quality of data resulting in more efficient survey analysis. This may be achieved by repeating a question with varied scale and then checking the consistency of relevant responses. A formal measure of inconsistency of a respondent is formulated in this work on the basis of his/her response to the same question repeated in multiple scales. The measure can be alternatively viewed as a measure of fuzziness attributed to respondent or attribute depending on its formulation. The probability distribution of this measure is obtained if the respondent marks completely at random. Undesirable respondents may be screened using the above mentioned probability distribution in the framework of statistical testing of hypothesis. The application extends to identifying fuzzy or unclear attributes along the similar lines. The paper also proposes a model based approach as well as another heuristic that can deal with screening inconsistent respondents and fuzzy attributes simultaneously.

Keywords: Attribute, Binomial test, fuzziness, market research, measure. probability distribution, psychometry. questionnaire, respondent and scale.

1 Motivation and Background Work

In survey research, it is not very rare to set few same or similar type of questions in the questionnaire. Indeed in behavioral as well as marketing research, often several 'items' are included to measure a single characteristic (often called a 'construct' in such case) like attitude, purchase intention etc. Even otherwise repetitions are done, although not frequently, to verify the authenticity, alertness. and seriousness of the respondent. However, one rarely varies the number of options in the above contexts or asks for rating the identical attribute/performance in multiple scales. The current work prescribes this to an extent, because it provides an opportunity to verify the clarity of thought and consistency of opinion of the respondent. Using this judiciously, one may be able to detect fake responses or respondents who are not providing sincere response. This should result in an increase in the efficiency of the results, even though the analysis would be based on less number of observations.

Repetition of questions always has a negative effect on the quality of genuine response. Thus one needs to exercise caution while taking such a step. Change of scale (given number of options against a question) helps in possibly disguising the repetition process and it may become less obvious and irritating to the respondent as a result of it. Of course, detection of inconsistency is less than obvious in such a situation, and this is precisely where the current work hopes to contribute. There is a quantum of literature (e.g. [8]) in psychometry which deals with reliability and consistency of response among items corresponding to a construct. However various measures of internal consistency like Cronbach's alpha, coefficient of beta (see [3], [7], among others) are not applicable in the given context since it is assumed here that a single question (item) is enough to retrieve information regarding the attribute(s) in concern, and any repetition is certainly not necessary from the measurement point of view. It is a different issue altogether that for certain attributes or constructs, a single-item question may not be adequate (viz. [9]).

In a few other circumstances, it may be necessary to seek responses in multiple scale. For example, in [4], a data set comprised of 58 respondents rating 24 attributes each in 5 scales containing 3,5,7,9 and 11 options. Such an enormous degree of repetitions were done by design for the specific purpose of investigating the quantum of fuzziness that can be attributed to the available number of options.

Adequate care was taken in choosing the settings and attribute to ensure that the respondents were motivated and encouraged to provide the ratings on multiple scales. Similar types of large multiple-scale survey data have been cited in [2], although they were motivated by different considerations. [4] proposed a quantification of the fuzziness as a function of the number of options available, pointing towards direction in determining the appropriate number of levels. It also acknowledged the dependency of the fuzziness in such human responses on the clarity of thought process of the respondent as well as on the nature of the attribute being rated, although no measurement of this was proposed. The measure that is put forward here, can also be thought of as an attempt to quantify that type of vagueness, and accordingly one may like to refer to it as a measure of fuzziness attributed to respondent or attribute, depending on its formulation. However, the term *inconsistency measure* will be predominantly used since it quantifies the lack of coherency (across different scales) in response. The details of the measure in terms of its derivation and motivation is given in Section 2.

Computational results from evaluation of the inconsistency measure on the cited data set is presented along with preliminary inference in Section 3. The derivation of the probability distribution of $\mathcal{I}$ (under completely random response) is described in Section 4, while the probability tables are included in the Appendix A. This measure is naturally specific to the respondent as well as attribute being rated or the question being asked. So if an attribute has significantly high fuzziness measure more or less consistently across all respondents, we may naturally infer that the attribute itself is fuzzy or unclear. Few formal approaches to facilitate the process of screening undesirable respondents and questions or attributes are described in Section 5. In Section 5.1 a class of typical testing of hypothesis is adopted using the probability distribution in Section 4. While the probability tables are valid for the extreme case of completely random marking of a respondent, the natural expectation is that the illustrated method would be also useful to detect less extreme kind of irregularities. In Section 5.2 a model-based approach is attempted for simultaneous removal of poor attributes and respondents. A general algorithm towards the same objective is presented in Section 5.3. Finally in conclusion, Section 6 provides a summary, direction for future research, along with few relevant remarks.

The proposed measure and technique is directly applicable when the data is in the rating scale. But it is also applicable with the Likert type scales when verbal descriptions are possibly attached with the alternatives, in which case these are later on converted into numerical scale at the data processing stage. Of course, with this later case, the typical problem of interpreting the data as of interval type remains. That criticism is also valid partially for the rating scale, which strictly speaking, provides only a ordinal type data.

2 Proposed Measure

To measure the consistency or the lack of it of ratings given in different scale by the same respondent on identical attribute, each response is converted into a 'common' scale, namely [0,1]. The basic philosophy behind this transformation is as follows. By giving a rating of i out of k in the integer scale, a respondent is expressing a rating of somewhere between $\frac{i-1}{k}$ and $\frac{i}{k}$ out of 1, in the continuous scale [0.1]. In this work, this unobservable (ideal) rating in the continuous scale [0,1] will be referred to as *score*. Thus any specific response or rating provides an information regarding this score through an interval. When a respondent gives any subsequent ratings of the same attribute on different scales, another interval that corresponds to the same score, is obtained. Ideally all these intervals associated with a single score (rating of a given attribute given by a fixed respondent) should overlap and lead to a more refined information regarding this score. However, if there is no common overlap between the intervals corresponding to the same score it reflects some degree of inconsistency or fuzziness of the respondent regarding the attribute in question. Level of overlapping of the intervals reflects the corresponding quantum of consistency in response. With that in mind, the measure is defined in terms of the maximal number of overlaps, or equivalently the modal frequency of the frequency (type) distribution arising out of transforming each response to a common scale.

An example would illustrate the above more clearly. Suppose, a respondent gives ratings 1 out of 2, 2 out 3 and 2 out of 5 to an attribute. On the (transformed) continuous (0.1) scale, the first rating indicates that, (s)he would give a score that lies somewhere between 0 and $\frac{1}{2}$. Similarly. the second rating corresponds to a score belonging to the interval $[\frac{1}{3}, \frac{2}{3}]$, and the third one to the interval $[\frac{1}{5}, \frac{2}{5}]$. Thus, a score of anywhere in the interval $[\frac{1}{3}, \frac{2}{5}]$ is consistent with all the three ratings given. and hence it may be said that this interval has a *frequency* of 3. Whereas, the interval $(0, \frac{1}{5})$ is consistent only with the first rating and hence has frequency 1; similarly the intervals $[\frac{1}{5}, \frac{1}{3})$, $(\frac{2}{5}, \frac{1}{2}]$, $(\frac{1}{2}, \frac{2}{3}]$, and $(\frac{2}{3}, 1)$, have frequencies 2, 2, 1 and 0 respectively. This can be summarized in the form of following *frequency distribution*:

range	$(0, \frac{1}{5})$	$[\frac{1}{5}, \frac{1}{3})$	$[\frac{1}{3}, \frac{2}{5}]$	$(\frac{2}{5}, \frac{1}{2}]$	$(\frac{1}{2}, \frac{2}{3}]$	$(\frac{2}{3}, 1)$
frequency	1	2	3	2	1	0

In the passing, it may be noted that the terminology 'frequency distribution' has a connotation which is slightly different connotation from the standard statistical literature, since the sum of the frequencies does not equate to the total number of observations on which it is based. But as long as the frequencies are used as weights, statistical measures like mean, mode, or standard deviation, computed by using the standard formulae, continue to have their usual interpretations.

As motivated earlier, the measure proposed in this work is based on the frequency of the modal class in a grouped data situation. In the numerical example given above, since any score [rating in the continuous scale (0,1)] in the range $[\frac{1}{3}, \frac{2}{5}]$ is consistent with all the ratings provided by the respondent, the inconsistency measure of this respondent should be 0. On the other hand, if the last rating was 4 (in stead of 2) out of 5, then one would have concluded that there is no score which is consistent with all the three ratings, and hence some positive inconsistency value should be attached to the respondent rating the corresponding attribute.

While the above-mentioned frequency distribution is conceptually appealing and simple to compute in small problems, it is relatively difficult to determine formal mathematical expressions of the (boundaries of the) intervals which compete for the modal class since the final interval are obtained by considering the overlaps of the generated intervals. Hence, an alternative rout is taken in terms of the *frequency function*

$$f_{\mathbf{x}}(t) = \sum_{i=1}^{p} I_{[\frac{x_i-1}{k_i}, \frac{x_i}{k_i}]}(t), \quad t \in [0, 1] \tag{1}$$

which is defined on the range of scores [0,1], where the ratings under consideration are x_i out of k_i, for $i = 1, 2, \ldots, p$, (with $k_1 < k_2 < \ldots < k_p$, w.l.o.g.) and $I_A(\cdot)$ is the indicator function of an interval A. The frequency function clearly depends on the response vector $\mathbf{x} = (x_1, \ldots, x_p)'$, and this is reflected in the suffix. The frequency function uniquely determines the frequency distribution and vice versa, and thus the two approaches are equivalent.

Note that the maximum possible value of the modal class frequency and equivalently the maximum attainable value $\max_t f_{\mathbf{x}}(t)$ is p, which corresponds to the case of having perfect consistency among all the p rating. The opposite extreme case happens if no two of the intervals overlap or equivalently $\max_t f_{\mathbf{x}}(t)$ is 1, signifying the highest order of inconsistency. The intermediate values of the maximum number of overlapping intervals signify the other possible levels of inconsistency. This motivates defining the measure of inconsistency $\mathcal{I}$ as:

$$\mathcal{I} = \left(1 - \frac{\max\limits_{t\in(0,1)} f_{\mathbf{x}}(t)}{p}\right) \times \frac{p}{p-1}. \tag{2}$$

Note that the $\frac{p}{p-1}$ term is incorporated in (2) only to ensure the maximum possible value of $\mathcal{I}$ to be 1, independent of the number ratings (p). Note that while this corresponds to the most haphazard, inconsistent or fuzzy rating pattern, similarly the minimum value of $\mathcal{I}$ is 0, which corresponds to the most clear rating pattern from the respondent. This minimal value of $\mathcal{I}$ is attained if and only if there is at least one score (x) which is consistent with all the ratings given by the respondent on the repeated attribute. These characterizations of the upper and lower limit of $\mathcal{I}$ is quite attractive in terms of standardizations; this facilitates its use in the comparison of the inconsistency under different situations. A minor limitation of $\mathcal{I}$ is that for certain combination of replicated scales (for example, if the attribute measurement is repeated thrice with the number alternatives being 3,4 and 5), $\mathcal{I}$ can not possibly attain the maximal value 1. It may be tempting to rescale the measure in such a situation. However since the issue is relatively minor and it appears that such modifications would bring in substantial complication, it is deemed preferable not to undertake such changes in the definition of the measure.

From the above proposed measure of fuzziness or inconsistency, one can arrive at a measure of consistency as:

$$\mathcal{C} = 1 - \mathcal{I}. \tag{3}$$

Since $\mathcal{I}$ is obtained by aggregating responses from all scale, it is free of scales and hence specific only to the respondent and the attribute being rated. Typically, in deriving $\mathcal{I}$ one can aggregate responses from all the respondents on the same attribute only if the respondents are expected to be homogeneous. Such a situation may arise when several judges are evaluating some performances; while $\mathcal{I}$ is not tailor-made to address this problem, it could be applicable and it might be of interest to compare it with Cohen's κ ([1]), Perreault and Leigh's measure ([6]) and Rust and Cooil's PRL measure ([8]). Conversely it would make sense to aggregate relevant items only if they correspond to measuring the same characteristic, like in a multi-item construct. However, it should be emphasized that the main scope of application of $\mathcal{I}$ is beyond either of the two above situations.

3 Preliminary Analysis

As seen in the relatively simple example of the previous section, the computation involved in $\mathcal{I}$ is pretty complex. The relevant C-programming routine is included in Appendix B. Data analysis from two studies, one fairly big and the other small, are included here, while the supporting material is included in Appendix C.1 and C.2 respectively. The first data set is reported in the earlier work [4]; it consists of 58 respondents rating 4 players (AJ, ST, SG, and AK) on six aspects of the game, namely fielding, sincerity, ability to face adverse situation, batting, bowling, and captaincy (24 attributes in total) each in 3,5,7,9, and 11-point scales.

The inconsistency of each respondent for each of the 24 attributes are shown in the double array format in Table 1 of Appendix C.1. The mode ratings are given in Table 2. The last two rows reflect the aggregate results. The row corresponding to *comb.* refers to the fuzzy measures obtained from the frequency distribution arising from combining all respondents. The last row are the column means. To highlight the inconsistency associated with each respondent and each attribute, Tables 3 and 4 are presented respectively with the first showing the inconsistency distribution of each respondent on 24 attributes and the second providing the distribution for each attribute on 58 respondents. For example, the respondent 1 has inconsistency measure 0 in 9 out of the 24 attributes, measure of 0.5 in 12 of them, and 0.75 in the remaining 3. Means of these distributions are also given for easier comparison.

Since the main entries in the matrix of Table 1 are obtained from five observations each (i.e. p in (2) is 5), the possible values (of $\mathcal{I}$) are 0, 0.25, 0.5, 0.75, and 1. All these, with the exception of 1, can be found in the table. Thus, no respondent is totally fuzzy for rating any attribute. At the same time, nobody is perfect either, i.e., everybody has some positive fuzzy measure for at least one (actually three) attribute. The overall fuzziness associated with the different attributes may be measured partially through the second last row of Table 1, which are obtained by computing (2) from respective frequency distributions pooled by combing all the respondents. This gives only a partial idea regarding fuzziness associated with the attribute, since the respondents are not necessarily homogeneous in the sense of having the same opinion of the attributes. A better reflection may be obtained by noting the mean fuzziness of the respondents for the attribute in question; this is reported in Table 4 as well as the last row of Table 1.

Table 2 gives the mode ratings reflecting the average measure of the attribute provided by the respondent. This is not pursued in any details here since the current work is concerned with associated fuzziness, not on the rating itself. However, mode is preferred to mean because of the basic philosophy of the measure in the given case.

The comparison among the respondents regarding inconsistency is brought out more clearly in Table 3. It would be an interesting exercise to study *similarity* between respondents; but with the focus of the current work in mind, that is not taken up here. Respondent No. 6, 12 and 53 have decidedly higher degree of consistency whereas Respondent No. 22, 2 and 41 may compete for being most confused or fuzzy. One may compare simply the exhibited mean of the fuzzy distributions for these comparative analysis, although inspecting the entire distribution is advisable.

A close look at Table 4 shows that the two attributes Batting-ST and Fielding-AJ stand out as being least fuzzy. Same conclusion will be arrived at by examining the outcomes by aggregating all respondents; however, the aggregate numbers are not that meaningful. It is perhaps not coincidental that these are the two attributes which received marked higher ratings than the other attributes (see Table 2). It is conceivable that the attributes receiving uniformly very poor ratings are also very consistent. but none of the attributes under current consideration qualifies for that. The evidence is much more compelling through $\mathcal{I}$, as can be seen in Table 4. A more objective way of interpreting the findings in Table 3 and 4 are presented in Section 5.

The second survey data comprised of responses from 10 respondents on three questions each repeated on four scales containing 3, 5, 7 and 9 options. The summary of results from this is included in Appendix C.2.

4 Probability distribution of $\mathcal{I}$

An ideal survey situation is reached when the attributes are devoid of any ambiguity, and the respondent are knowledgeable as well as honest. In such a case, the inconsistency measure $\mathcal{I}$. as proposed, should be equal to zero. The other extreme situation is when the respondent is marking totally at random without paying any attention to what is being asked for. While such a situation may be not very common, even an approximation of this may cause disaster for any decision made on the basis of such data; and thus, it would be of paramount importance to detect such responses. To move in this direction, the probability distribution of $\mathcal{I}$ under completely random ticking is derived here. This is used in appropriate statistical testing of hypothesis as outlined in the next section.

The basis of obtaining these distributions is complete enumeration of all possible sets of responses. Under completely random response, all these possible combinations are to be assigned equal probability. Thus, one needs to compute the proportion of response sets which lead to the inconsistency measure equal to $\frac{i}{p-1}$ to arrive at the $\mathbb{P}[\mathcal{I} = \frac{i}{p-1}]$, for $i = 0, 1, \ldots, p-1$. In the above, p stands for the number of scales, and i stands for the maximal number of overlapping intervals the response sets lead to as a result of their conversion to [0,1] scale. In describing the derivation of the probability distribution, the characterization of the various values of $\mathcal{I}$ would be found to be useful. Additionally it may also provide insight to the measure.

For example, there is a perfect consistency if and only if the intervals generated from the repeated measurements have some common overlap, i.e.

$$\mathcal{I} = 0 \Longleftrightarrow \max_{1 \le i \le p} \frac{x_i - 1}{k_i} \le \min_{1 \le i \le p} \frac{x_i}{k_i}.$$

The other extreme case is when the inconsistency is of the highest order which can result only from all the intervals being disjoint, i.e.

$$\mathcal{I} = 1 \Longleftrightarrow \left(\frac{x_{i_1}}{k_{i_1}} < \frac{x_{i_2} - 1}{k_{i_2}}\right) \cap \left(\frac{x_{i_2}}{k_{i_2}} < \frac{x_{i_3} - 1}{k_{i_3}}\right) \cap \ldots \cap \left(\frac{x_{i_{p-1}}}{k_{i_{p-1}}} < \frac{x_{i_p} - 1}{k_{i_p}}\right),$$

for some permutation $\{i_1, i_2, \ldots, i_p\}$ of $\{1, 2, \ldots, p\}$. The characterization of the middle values of $\mathcal{I}$ are more difficult. This is achieved by looking at the cumulative values in the following way:

$$\mathcal{I} \leq \frac{1}{p-1} \Longleftrightarrow \max_t f_{\mathbf{x}}(t) \geq p-1 \Longleftrightarrow \bigcup_{i=1}^{p} \Big(\max_{\substack{1 \leq j \leq p \\ j \neq i}} \frac{x_j - 1}{k_j} \leq \min_{\substack{1 \leq j \leq p \\ j \neq i}} \frac{x_j}{k_j} \Big),$$

$$\mathcal{I} \leq \frac{2}{p-1} \Longleftrightarrow \max_t f_{\mathbf{x}}(t) \geq p-2 \Longleftrightarrow \bigcup_{i=1}^{p} \bigcup_{\substack{l=1 \\ \neq i}}^{p} \Big(\max_{\substack{1 \leq j \leq p \\ j \neq i,l}} \frac{x_j - 1}{k_j} \leq \min_{\substack{1 \leq j \leq p \\ j \neq i,l}} \frac{x_j}{k_j} \Big),$$

and so on.

Appendix A contains the probability distribution tables for p up to 5, with k_i's ranging up to 11. these should cover most frequently used cases. Using the characterizations given above, the other cases may be generated as required. A C-program routine is available with the author to enact this.

5 Screening out Respondents and/or Attributes

From a matrix or double-array of inconsistency measures (with respondents being in one array (row) and attributes in the other, column), the objective is to identify the significantly inconsistent respondent(s) and attribute(s). The probability distribution of the inconsistency measure is known if the response is given completely arbitrarily, i.e. by randomly picking one alternative from the given options. On the other hand, for the ideal case, the probability distribution is degenerate at zero. Of course, as usual, the most typical cases would be somewhere in between the two; but it may be possible to make the necessary critical decisions based on these two extreme situations.

The simplest approach is a one-way approach. While deciding whether any respondent should be thrown out, generate the frequency distribution considering the inconsistency values for that respondent corresponding to all the attributes. This may be compared with the probability distributions under the two hypotheses. (The reverse would be done if one were to decide whether a particle attribute should be discarded). There may be several ways to implement these, which we illustrate through the cricket survey data set in the next subsection.

An alternative model-based approach, which possibly does not suffer from this, is illustrated in the following subsection. Finally in Section 5.3, an heuristic for simultaneous screening of respondents and attributes are presented.

5.1 Approach I: Unidimensional Method in Testing of Hypothesis Framework

To begin with, frame the null hypothesis as the respondent ticking completely at random against the alternative of the response being authentic; i.e.

$$H_0: \quad p = p_0 \qquad \text{vs.} \qquad H_1: \quad p \gg p_0, \tag{4}$$

where p is the probability of the inconsistency measure taking the value 0 (or $\leq$ any specified value) and p_0 may be obtained from Appendix A. The value of p under the alternative is ideally 1; however to avoid degeneracy, one may set it at a large value p^* near 1. It is expected that in the case genuine response, there will be overwhelming evidence against the null hypothesis, given sufficient opportunity (or replication). In the case of a question being repeated more than two scales, there is a further choice to be made regarding the test criterion, because there are possible values of the inconsistency measure other than 0 and 1. For example, in our data set, the ratings were sought in scales 3,5,7,9,11. So while testing the validity of a respondent, the test statistic could be the number of inconsistency measures (for the 24 attributes) equal to 0, or $\leq \frac{1}{4}$, or $\leq \frac{1}{2}$ or $\leq \frac{3}{4}$. Of course, the last one is not quite appropriate because even under the above null hypothesis, $\mathbb{P}[\mathcal{I} \leq \frac{3}{4}] = 1$ (as is obviously the case under the alternative). The test is one-sided, with the null hypothesis being rejected for large values of the relevant test statistic. The cut-off values may be obtained from the fact that the null distribution is Binomial with the respective probabilities to be obtained from the table given in Appendix A. For the first numerical example, the null hypothesized value will thus be 0.0032, 0.069, and 0.5013, respectively, for the three choices and the cut-off points are 2, 6, and 19 ($\alpha = 0.003, 0.005, 0.003$) respectively for the three possible test statistics. Not surprisingly (considering the interest of the respondents in the chosen topics), all the null hypotheses are rejected even for these small choices of α, and hence all the respondents may be considered be providing conscious response. The same turns out to be the decision while verifying the consistency for all the attributes. It may be argued that the above method of testing does not make full use of the available data set and a more powerful test may be designed comparing the empirical distribution itself with the null hypothesized one. Thus one may use the Kolmogorov-Smirnov or Chi-square goodness-of-fit test. The problem with these are that from the rejection of the null hypothesis of completely random ticking, it may not be valid to conclude the authenticity of the response.

Now consider switching the null and the alternative hypothesis. This may be in order specially if not many questions are repeated and hence there is not enough opportunity for the respondent to produce evidence against completely random ticking. In such a case, it may be proper to assume that the respondents (attributes) are genuine by default (null hypothesis) and look for possible evidence against it. However, since the (ideal) null distribution is degenerate in this case, any single positive inconsistency would lead to the rejection of the null hypothesis. This, however, seems a little too harsh. A solutions to this would be to give some allowance, which admittedly suffers from arbitrariness. For example, one possible framework is to test

$$H_0 : \mathbb{P}[\mathcal{I} = 0] \geq p^\star \qquad \text{vs.} \qquad H_1 : \mathbb{P}[\mathcal{I} = 0] = p_0 \quad (\text{or } < p^\star). \tag{5}$$

where $p^\star$ would be the pre-set high value. The following table gives the cut-off values of the test-statistics (Binomial with $n = 24$) for α closest to but not exceeding 0.05, using the non-randomized tests:

	$p^\star$			
	0.75	0.8	0.9	0.95
c	13	15	18	20
(α)	(.021)	(0.036)	(0.028)	(0.030)

Now, we can refer to the Table 3 for detailed inference in the various cases. For example, only respondent no. 6 remains valid if we use the stringent choice of $p^\star = 0.95$, where as for $p^\star = 0.75$, respondent no 8, 12, 17, 48, 49, 52, 53 also are valid.

Like before, there is a case for considering other null hypotheses like

$$H_0' : \mathbb{P}[\mathcal{I} \leq \frac{1}{4}] \geq p^\star$$

in this framework. Naturally, more respondents will be admissible in this setup. For example, a reference to Appendix E and Table 3 of Appendix C.1 concludes that for the above hypothesis and $p^\star = 0.75$, Respondents 1, 5, 15, 22 and 41 would be discarded.

The use of the probability distribution of $\mathcal{I}$ derived in Section is clear from the above description. It allows to have the null distribution in the straight formulation (4), which is critical in terms of test construction. In the reverse formulation (5), it can provide the power of the test.

Illustration of all the above approaches on the smaller data set is included in Appendix C.

5.2 Approach II: A Product Model Based Method

In this subsection we want to model the inconsistency pattern exhibited by a groups of respondent on a bunch of attributes. Towards that goal, consider the following framework. Let

$$Y_{ij} = \begin{cases} 1 & \text{if respondent } i \text{ is non-fuzzy or consistent for attribute } j \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

Let $P_{ij} = P[Y_{ij} = 1]$. Then P_{ij}'s should be high (close to 1) for 'good' respondent and attribute pairs. Conversely, it may be small (close to 0) either because of the fuzziness of the attribute or because of the inconsistency of the respondent. This motivates formulation of the following product model:

$$P_{ij} = p_i q_j, \tag{7}$$

where p_i is the consistency index of respondent i, and q_j is the un-ambiguity associated with attribute j. Thus, P_{ij} could be small either because p_i is small, or because q_j is small. The next step is to estimate the $n + l$ values p_i's and q_j's, based on $n \times l$ Y-values. The final objective, of course, is to identify fuzzy respondents and/or attributes, i.e. the ones associated with low p_i's or q_j's.

One natural set of estimators are:

$$\hat{p}_i = \frac{1}{l}\sum_{j=1}^{l} Y_{ij}, \quad \hat{q}_j = \frac{1}{n}\sum_{i=1}^{n} Y_{ij}.$$

Unfortunately, they underestimate their respective targets, as

$$\boldsymbol{E}(Y_{ij}) = P_{ij} = p_i q_j \Longrightarrow \boldsymbol{E}(\hat{p}_i) = p_i \bar{q}, \tag{8}$$

where $\bar{q} = \frac{1}{l}\sum_{j=1}^{q} q_j$ which is ≤ 1, with inequality holding unless $q_j = 1$, $\forall j$. However, (8) also implies that the relative values of $\hat{p}_i$'s may still be empirically used to determine which respondents are very inconsistent (or corresponds to small values of p_i). In fact, since

$$\bar{\hat{p}} = \frac{1}{l}\sum_{i=1}^{n} \hat{p}_i = \sum_{j=1}^{l} \hat{q}_j = \bar{\hat{q}}$$

and one can use

$$\frac{\hat{p}_i}{\bar{\hat{p}}}$$

as approximately unbiased estimator of

$$\frac{p_i}{\bar{p}}.$$

However, a statistically appealing scheme regarding possible discard of respondents or attributes seems to be elusive.

Results from implementing the above procedures on the cricket survey data is included in Appendix D. If we set (albeit arbitrarily) a value of $\frac{1}{2}$ as a threshold value for screening respondents and attributes based on relative proportions (i.e., $\hat{p}_i/\bar{\hat{p}}$ or $\hat{q}_j/\bar{\hat{q}}$), then Respondents 14, 22 and 41 would be deleted as well as the attribute reflecting AK's ability in adverse situation.

Note that one can easily modify the binary transformation (6) in order to give weightage to fractional inconsistency values.

5.3 An Heuristic

A limitation of the unidimensional testing of hypothesis in Section 5.1 is that one may end up rejecting an attribute because of several attributes being fuzzy. The product model approach in 5.2 attempts to overcome this, but it is not clear if the estimated parameters help achieve this goal. In this subsection we describe an algorithm to remove inconsistent attributes and fuzzy attributes in a single procedure, but in successive steps, keeping in tune with the average inconsistency or fuzziness values associated with them. It can also be used in conjunction with the methods described in Section 5.1 and 5.2. This helps in implementing simultaneous screening of attributes and respondents, which is important because it ensures that the poor response from discarded respondents (on attributes) would not affect the decision on discarding attributes (respondents). Because of the graded removal scheme, one can stop at any intermediate stage depending on the tolerable label of inconsistency or fuzziness.

The process can be implemented using either the consistency response matrix or the inconsistency response matrix and the two approaches are equivalent. In the following, the former will be used for illustration, and hence small row/column averages would ask for removal in our setup. Additionally, the data matrix is converted to binary type by identifying perfect consistency with 1 and its negation with zero. This is not necessary in general, and the interpretation and implication varies to a small extent depending on whether this conversion is done or not; more discussion on this follows later on. The steps to be followed are as follows:

1. Compute the consistency of each respondent for each attribute in a matrix form. Decide on whether to use this or its binary form by converting any fractional consistency to zero as in (6). Note that such a conversion gives equal importance to all the different magnitudes of inconsistencies.
2. Compute the row and column averages. Decide on the critical index so that if an average consistency falls below this index, then the corresponding respondent or attribute may call for rejection. One may like to set a row critical index which is different from the column critical index.
3. Choose the smallest row or column average(s); if this value is less than the critical index, then drop the respective row or column. In case of a tie, all the relevant rows (or columns) may be removed at the same stage; but a row and a column should not be removed at the same stage.
4. Recompute the row (column) averages if one or more columns (rows) have been removed in the previous stage. Repeat the process of screening till all the averages are above the critical index.

Following is an illustration of the above algorithm using the second (small) survey data described in Section 3. The binary conversion of the data was performed and a critical value was not set apriori to provide complete demonstration. Also, attribute and respondent consistency was given equal importance, so that row/column corresponding to lowest average consistency was deleted at each stage.

Step-wise detection of fuzzy attributes and inconsistent respondents

from Response Consistency Matrix

		Question No			average consistency			
	Respondent No.	Q1	Q2	Q3	stage 0	stage 1	stage 2	stage 3
	1	0	0	0	0	x		
	2	1	0	0	$\frac{1}{3}$	$\frac{1}{3}$	x	
	3	1	1	1	1	1	1	1
	4	0	0	0	0	x		
	5	1	1	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	1
	6	1	1	1	1	1	1	1
	7	1	1	1	1	1	1	1
	8	1	1	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	1
	9	1	1	1	1	1	1	1
	10	1	1	0	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	1
average consistency	stage 0	$\frac{8}{10}$	$\frac{7}{10}$	$\frac{4}{10}$	$\frac{19}{30}$			
	stage 1	1	$\frac{7}{8}$	$\frac{4}{8}$		$\frac{19}{24}$		
	stage 2	1	1	$\frac{4}{7}$			$\frac{18}{21}$	
	stage 3	1	1	x				1

In the first stage, respondent 1 and 4 are 'removed' (and marked by 'x' against respective rows), since they are inconsistent for all the three questions. After their removal, column averages are re-computed, while naturally, remaining row averages stay the same. At the next stage, the minimum row average is for respondent 2 and the minimum column average is for Q3; because we chose to have equal importance for row and column, the former gets removed because its average consistency (1 out of 3) is the lower. Again, the column averages are recomputed, but this time the average consistency for Q3 (4 out of 7) is smaller than the average consistency of any of the remaining respondents (No's 3, 5 — 10). Hence at stage 3 Q3 is removed. The process culminates here since all the remaining seven respondents are consistent for the remaining questions (Q1 and Q2).

Note that, if we had apriori set the critical index to be $\frac{1}{2}$, for example, the algorithm would have stopped at stage 2 itself, leaving all three attributes along with the seven respondents. One the other hand, if the row (respondent) critical index is lowered to $\frac{1}{4}$, while the column (attribute) critical index stays at $\frac{1}{2}$, then at stage 2, Q3 would be deleted and the algorithm will stop at that (Respondent 1 and 4 already removed). An alternative procedure for termination of the algorithm could be set by specifying an overall acceptable level of consistency, in which case one can refer to the bottom right corner of the above table. For example, if one had set this at 75% level, then the stopping rule would have prompted to stop after stage 1 itself, while for the 90% level, one needed to cover all the three stages.

The significant of the critical index would have been different had the binary transformation been not performed. Using the transformation, a critical index of $\frac{1}{2}$ implies that a respondent who is perfectly consistent for every 1 out of 2 attributes (or better) is acceptable. Such interpretation would not be valid if the transformation was not performed, and one would require to interpret only in terms of average consistency.

6 Summary and General Discussion

6.1 Estimating the Central Tendency of Score

Measurement of the same attribute in multiple scale has been advocated in this work only for the purpose of the validation. It is neither necessary nor recommended for all even many items or attributes. However, in the case of such a repetition, it is natural to expect that a more refined estimate of a measure of central tendency of the attribute would be available based on the richer collection of data. Based on the motivation of score conversion between the scales, as described in Section 2, the mean of the modal class may be taken as central rating (score) given by the respondent, in the transformed scale (out of 1). In case of there being more than one modal class, the average of the lower boundary of the smallest modal class and the upper boundary of the highest modal class should be taken. The later is different from the standard practice of computing mode from the grouped data; but it makes better sense in light of the logic behind the development of $\mathcal{I}$.

6.2 Recommended Level and Nature of Repetition

As mentioned already, it is very important not to get carried away with the concept of repeating similar question, with or without the scale varying. An increase in length of the questionnaire has an adverse effect on the response rate. In addition, even when response is obtained, the increasing risk of irritating the respondent may hurt the quality of response. Thus the principle of repeating question can be prescribed only to a limited extent and appropriate caution need to be taken along with it. If the scale is not changed with each repetition, while inconsistencies become more obvious to catch, the above mentioned hazard becomes much more prominent.

What choices of k_i's can one recommend? Is it better to repeat two attributes each twice, or a single attribute three or four times? These are some of the practical questions which need to be resolved with a mixture of theoretical and practical considerations. For a fully conscious respondent $\mathcal{I}$ is degenerate at 0, and hence the smaller is the probability of $\mathcal{I} = 0$ under random ticking the better it would be from the testing of hypothesis point of view. A closer inspection of the probability tables in Appendix A reveals the following characteristics of $P(k_1, \ldots, k_p) = \mathbb{P}[\mathcal{I} = 0]$:

- $\min_{k_1,\ldots,k_p} P(k_1, \ldots, k_p)$ (with p fixed) is a decreasing function of p.
- Co-ordinate-wise the function $P(\cdot)$ is *typically* non-increasing, provided one separates the odd and the even sequence; i.e.

$$\{P(2m, k_2, \ldots, k_p), \quad m \geq 1\} \qquad \text{and} \qquad \{P(2m-1, k_2, \ldots, k_p), \quad m \geq 1\}$$

  are two (essentially) non-increasing sequences (with k_i's fixed). Note that there are few minor exceptions to the above, for e.g.

$$P(3,7) < P(3,9), \qquad P(3,4,7) < P(3,4,9),$$

  which are possibly linked with relative primeness of the arguments, and may be of separate academic interest. But, with the current focus of the study, this has not been explored further.

The first property suggests that the larger is the number of questions being repeated, the better it is in terms of catching consistencies. This is of course obvious, even intuitively, but at clear conflict from practical considerations. The second property suggests high number of options. However, traditional guidelines and earlier research (viz. [4], [5]) suggest limiting each scale to between five and nine. Finally, note that $[P(8,9)]^2 = .0493$ which is marginally higher that $P(7,8,9) = .0437$. The author is of the opinion that disguising repeat of two questions (each twice) is easier than repeat of a single question three times in a moderately large questionnaire. Hence the former would be useful as both would lead to roughly similar consistence response by pure accident.

6.3 Summary and Future Research

In survey questionnaire, it is typically a norm to have the identical scale for all the questions and not to repeat any question. The main recommendation of this work contradicts both of these by suggesting repetition of one or few questions on multiple and varying scales. Different scales while repeating may help disguise the act of repeating which by itself may help to catch false or insincere responses. A measure of inconsistency has been framed in this work whose distribution under random response has also been obtained. The work concludes with three formal or semi-formal methods of screening out respondents and attributes on the basis of the derived measure.

As mentioned earlier, validation of attributes is a topic of considerable interest in social and behaviorial studies involving multi-item contrasts. Consistency between such items have been studied in the psychometric literature through various measures, most notably through Krochbach's alpha and the beta coefficients. While there is some similarity of objective between the present study and that of cited works, a fundamental assumption in our framework is that there is no measurement problem of the attribute and hence a single item is enough to record the relevant response. Having noted that, the measure proposed in the current work may possibly play a role in those situations as well. However the relevant experimentation followed up by the comparative performance of $\mathcal{I}$ with the traditional measures is yet to be done and the author plans to pursue that.

In the framework of Section 5.1, yet another possibility is to test for

$$H_0 : \mathbb{P}[\mathcal{I} = \frac{i}{m-1}] \quad \text{is non-increasing function of } i, \tag{9}$$

against possible alternative. It is expected that a likelihood test using order restricted inference would be helpful in this regard; the details, however, is yet to worked out.

One can obtain the correlation matrix between the repeated scales, on the basis of responses of from all respondents on a fixed attributes. If the attribute is clear and all the respondents are authentic, one would expect the all the entries of this correlation matrix to be close to 1, and the closeness can be measured by 1 minus the average of the above matrix elements. However, if one or more respondents are inconsistent (for the given attribute), then dropping these respondents from the correlation matrix construction would lead to substantial rise to the average correlation. Similar could be the philosophy behind screening attributes. Detailed procedures implementing this philosophy is likely to be possible along the methods described in Section 5; we plan to attempt this in future.

It is not clear if progress in the model based approach in Section 5.2 can be made to achieve a formal or statistically valid decisions similar to Approach I. Alternative models in this regard may also be worthwhile. For example, similar to correspondence analysis or reciprocal averaging, one may target to have $(x_1, \ldots, x_n)$, and $(y_1, \ldots, y_l)$ satisfying:

$$x_i = \frac{\sum_{j=1}^{l} C_{ij} y_j}{\sum_{j=1}^{l} C_{ij}}, \quad i = 1, \ldots, n, \tag{10}$$

$$y_j = \frac{\sum_{i=1}^{n} C_{ij} x_i}{\sum_{i=1}^{n} C_{ij}}, \quad j = 1, \ldots, l, \tag{11}$$

provided the denominators in the right hand sides are not zero, otherwise, the corresponding left hand side expressions should be set to zero. Note that x_i's and y_j's play roles similar to p_i's and q_j's of Section 5.2. One can also think of using the binary Y_{ij}'s of (6) instead of the consistency values C_{ij}'s. Null rows or columns in C do not create any major concern because one may drop the corresponding respondent or attribute in such a case to begin with. It is well known that (10) and (11) does not entail any non-trivial (not vectors of ones) solutions and accordingly one actually looks for minimizing $\xi(\geq 1)$ to satisfy:

$$x_i = \sqrt{\xi}\frac{\sum_{j=1}^{l} C_{ij} y_j}{\sum_{j=1}^{l} C_{ij}}, \quad i = 1, \ldots, n, \qquad y_j = \sqrt{\xi}\frac{\sum_{i=1}^{n} C_{ij} x_i}{\sum_{i=1}^{n} C_{ij}}, \quad j = 1, \ldots, l. \tag{12}$$

However the applicability of the above model in the current problem is yet to be fully resolved.

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A Probability Tables of $\mathcal{I}$ Under Random Marking

For the probability distribution corresponding to other levels of repetitions (p) or scale combinations (k_i's), contact the author for relevant C Program routine. Probability distributions of the consistency measure C are essentially the same with the values transformed as is obvious from (3)

Appendix A

Probability Distribution of the Inconsistency Measure

Notation:

p ⇒ number of scales on the basis of which φ is computed

k_1, k_2, … ⇒ number of options in the different scales

Probability Table for p = 2

Scale		values of inconsistency		scale		values of inconsistency		scale		values of inconsistency	
k_1	k_2	0	1	k_1	k_2	0	1	k_1	k_2	0	1
2	3	0.6667	0.3333	4	6	0.4167	0.5833	6	13	0.2308	0.7692
2	4	0.7500	0.2500	4	7	0.3571	0.6429	7	8	0.2500	0.7500
2	5	0.6000	0.4000	4	8	0.4375	0.5625	7	9	0.2381	0.7619
2	6	0.6667	0.3333	4	9	0.3333	0.6667	7	10	0.2286	0.7714
2	7	0.5714	0.4286	4	10	0.3500	0.6500	7	11	0.2208	0.7792
2	8	0.6250	0.3750	4	11	0.3182	0.6818	7	12	0.2143	0.7857
2	9	0.5556	0.4444	4	12	0.3750	0.6250	7	13	0.2088	0.7912
2	10	0.6000	0.4000	4	13	0.3077	0.6923	8	9	0.2222	0.7778
2	11	0.5455	0.4545	5	6	0.3333	0.6667	8	10	0.2250	0.7750
2	12	0.5833	0.4167	5	7	0.3143	0.6857	8	11	0.2045	0.7955
2	13	0.5385	0.4615	5	8	0.3000	0.7000	8	12	0.2292	0.7708
3	4	0.5000	0.5000	5	9	0.2889	0.7111	8	13	0.1923	0.8077
3	5	0.4667	0.5333	5	10	0.3600	0.6400	9	10	0.2000	0.8000
3	6	0.5556	0.4444	5	11	0.2727	0.7273	9	11	0.1919	0.8081
3	7	0.4286	0.5714	5	12	0.2667	0.7333	9	12	0.2037	0.7963
3	8	0.4167	0.5833	5	13	0.2615	0.7385	9	13	0.1795	0.8205
3	9	0.4815	0.5185	6	7	0.2857	0.7143	10	11	0.1818	0.8182
3	10	0.4000	0.6000	6	8	0.2917	0.7083	10	12	0.1833	0.8167
3	11	0.3939	0.6061	6	9	0.2963	0.7037	10	13	0.1692	0.8308
3	12	0.4444	0.5556	6	10	0.2667	0.7333	11	12	0.1667	0.8333
3	13	0.3846	0.6154	6	11	0.2424	0.7576	11	13	0.1608	0.8392
4	5	0.4000	0.6000	6	12	0.3056	0.6944	12	13	0.1538	0.8462

Probability Table for p = 3

scale			values of inconsistency			scale			values of inconsistency		
k_1	k_2	k_3	0	1/2	1	k_1	k_2	k_3	0	1/2	1
2	3	4	0.3333	0.6667	0.0000	3	8	11	0.0758	0.6818	0.2424
2	3	5	0.2667	0.7333	0.0000	3	9	10	0.0815	0.7111	0.2074
2	3	6	0.3333	0.6667	0.0000	3	9	11	0.0774	0.7071	0.2155
2	3	7	0.2381	0.7619	0.0000	3	10	11	0.0667	0.6727	0.2606
2	3	8	0.2500	0.7500	0.0000	4	5	6	0.1167	0.7000	0.1833
2	3	9	0.2593	0.7407	0.0000	4	5	7	0.1000	0.6857	0.2143
2	3	10	0.2333	0.7667	0.0000	4	5	8	0.1125	0.7000	0.1875
2	3	11	0.2121	0.7576	0.0303	4	5	9	0.0889	0.6778	0.2333
2	4	5	0.2500	0.7500	0.0000	4	5	10	0.1100	0.6900	0.2000
2	4	6	0.2917	0.7083	0.0000	4	5	11	0.0818	0.6636	0.2545
2	4	7	0.2143	0.7500	0.0357	4	6	7	0.0952	0.6786	0.2262
2	4	8	0.2813	0.7188	0.0000	4	6	8	0.1146	0.6979	0.1875
2	4	9	0.1944	0.7778	0.0278	4	6	9	0.0926	0.6759	0.2315
2	4	10	0.2250	0.7500	0.0250	4	6	10	0.0917	0.6750	0.2333
2	4	11	0.1818	0.7727	0.0455	4	6	11	0.0758	0.6591	0.2652
2	5	6	0.2000	0.7667	0.0333	4	7	8	0.0893	0.6786	0.2321
2	5	7	0.1714	0.7714	0.0571	4	7	9	0.0714	0.6429	0.2857
2	5	8	0.1750	0.7750	0.0500	4	7	10	0.0714	0.6429	0.2857
2	5	9	0.1556	0.7778	0.0667	4	7	11	0.0649	0.6299	0.3052
2	5	10	0.2000	0.7600	0.0400	4	8	9	0.0764	0.6736	0.2500
2	5	11	0.1455	0.7818	0.0727	4	8	10	0.0813	0.6750	0.2438
2	6	7	0.1667	0.7857	0.0476	4	8	11	0.0682	0.6591	0.2727
2	6	8	0.1875	0.7708	0.0417	4	9	10	0.0611	0.6278	0.3111
2	6	9	0.1667	0.7778	0.0556	4	9	11	0.0556	0.6111	0.3333
2	6	10	0.1667	0.7833	0.0500	4	10	11	0.0545	0.6136	0.3318
2	6	11	0.1364	0.7879	0.0758	5	6	7	0.0762	0.6381	0.2857
2	7	8	0.1429	0.7857	0.0714	5	6	8	0.0750	0.6333	0.2917
2	7	9	0.1270	0.7778	0.0952	5	6	9	0.0741	0.6296	0.2963
2	7	10	0.1286	0.7857	0.0857	5	6	10	0.0800	0.6467	0.2733
2	7	11	0.1169	0.7792	0.1039	5	6	11	0.0606	0.6121	0.3273
2	8	9	0.1250	0.7917	0.0833	5	7	8	0.0643	0.6143	0.3214
2	8	10	0.1375	0.7875	0.0750	5	7	9	0.0603	0.6032	0.3365
2	8	11	0.1136	0.7955	0.0909	5	7	10	0.0686	0.6286	0.3029
2	9	10	0.1111	0.7889	0.1000	5	7	11	0.0545	0.5922	0.3532
2	9	11	0.1010	0.7778	0.1212	5	8	9	0.0556	0.5889	0.3556
2	10	11	0.1000	0.7909	0.1091	5	8	10	0.0650	0.6200	0.3150
3	4	5	0.1667	0.7333	0.1000	5	8	11	0.0500	0.5773	0.3727
3	4	6	0.1944	0.7222	0.0833	5	9	10	0.0578	0.6089	0.3333
3	4	7	0.1429	0.7381	0.1190	5	9	11	0.0465	0.5657	0.3879
3	4	8	0.1667	0.7292	0.1042	5	10	11	0.0509	0.6000	0.3491
3	4	9	0.1481	0.7407	0.1111	6	7	8	0.0595	0.5952	0.3452
3	4	10	0.1333	0.7333	0.1333	6	7	9	0.0582	0.5926	0.3492
3	4	11	0.1212	0.7273	0.1515	6	7	10	0.0524	0.5762	0.3714
3	5	6	0.1556	0.7333	0.1111	6	7	11	0.0476	0.5584	0.3939
3	5	7	0.1238	0.7238	0.1524	6	8	9	0.0556	0.5880	0.3565
3	5	8	0.1167	0.7167	0.1667	6	8	10	0.0542	0.5750	0.3708
3	5	9	0.1259	0.7259	0.1481	6	8	11	0.0455	0.5568	0.3977
3	5	10	0.1333	0.7200	0.1467	6	9	10	0.0481	0.5667	0.3852
3	5	11	0.1030	0.7152	0.1818	6	9	11	0.0438	0.5522	0.4040
3	6	7	0.1270	0.7460	0.1270	6	10	11	0.0394	0.5303	0.4303
3	6	8	0.1250	0.7361	0.1389	7	8	9	0.0437	0.5397	0.4167
3	6	9	0.1481	0.7407	0.1111	7	8	10	0.0429	0.5357	0.4214
3	6	10	0.1111	0.7333	0.1556	7	8	11	0.0390	0.5227	0.4383
3	6	11	0.1010	0.7273	0.1717	7	9	10	0.0381	0.5175	0.4444
3	7	8	0.0952	0.7024	0.2024	7	9	11	0.0361	0.5079	0.4560
3	7	9	0.1005	0.7196	0.1799	7	10	11	0.0338	0.4987	0.4675
3	7	10	0.0857	0.6952	0.2190	8	9	10	0.0361	0.5056	0.4583
3	7	11	0.0823	0.6926	0.2251	8	9	11	0.0328	0.4899	0.4773
3	8	9	0.0926	0.7130	0.1944	8	10	11	0.0318	0.4864	0.4818
3	8	10	0.0833	0.6917	0.2250	9	10	11	0.0283	0.4626	0.5091

Probability Table for p = 4

scale				values of inconsistency				Scale				values of inconsistency			
k_1	k_2	k_3	k_4	0	1/3	2/3	1	k_1	k_2	k_3	k_4	0	1/3	2/3	1
2	3	4	5	0.1000	0.5333	0.3667	0.0000	2	5	8	11	0.0273	0.3477	0.6250	0.0000
2	3	4	6	0.1250	0.5417	0.3333	0.0000	2	5	9	10	0.0311	0.3667	0.6022	0.0000
2	3	4	7	0.0833	0.5238	0.3929	0.0000	2	5	9	11	0.0242	0.3273	0.6465	0.0020
2	3	4	8	0.1042	0.5417	0.3542	0.0000	2	5	10	11	0.0273	0.3564	0.6164	0.0000
2	3	4	9	0.0833	0.5185	0.3981	0.0000	2	6	7	8	0.0357	0.3839	0.5804	0.0000
2	3	4	10	0.0833	0.5250	0.3917	0.0000	2	6	7	9	0.0317	0.3624	0.6058	0.0000
2	3	4	11	0.0682	0.5000	0.4318	0.0000	2	6	7	10	0.0310	0.3643	0.6048	0.0000
2	3	5	6	0.0889	0.5222	0.3889	0.0000	2	6	7	11	0.0260	0.3377	0.6320	0.0043
2	3	5	7	0.0667	0.4762	0.4571	0.0000	2	6	8	9	0.0324	0.3750	0.5926	0.0000
2	3	5	8	0.0667	0.4833	0.4500	0.0000	2	6	8	10	0.0354	0.3792	0.5854	0.0000
2	3	5	9	0.0667	0.4815	0.4519	0.0000	2	6	8	11	0.0265	0.3523	0.6193	0.0019
2	3	5	10	0.0733	0.4800	0.4467	0.0000	2	6	9	10	0.0278	0.3537	0.6167	0.0019
2	3	5	11	0.0545	0.4606	0.4848	0.0000	2	6	9	11	0.0236	0.3283	0.6431	0.0051
2	3	6	7	0.0714	0.5079	0.4206	0.0000	2	6	10	11	0.0227	0.3288	0.6439	0.0045
2	3	6	8	0.0764	0.5139	0.4097	0.0000	2	7	8	9	0.0238	0.3214	0.6508	0.0040
2	3	6	9	0.0802	0.5123	0.4074	0.0000	2	7	8	10	0.0250	0.3304	0.6411	0.0036
2	3	6	10	0.0667	0.5000	0.4333	0.0000	2	7	8	11	0.0211	0.3084	0.6656	0.0049
2	3	6	11	0.0556	0.4798	0.4646	0.0000	2	7	9	10	0.0206	0.3032	0.6698	0.0063
2	3	7	8	0.0536	0.4583	0.4881	0.0000	2	7	9	11	0.0188	0.2886	0.6840	0.0087
2	3	7	9	0.0529	0.4550	0.4921	0.0000	2	7	10	11	0.0182	0.2896	0.6844	0.0078
2	3	7	10	0.0476	0.4429	0.5095	0.0000	2	8	9	10	0.0208	0.3083	0.6653	0.0056
2	3	7	11	0.0433	0.4286	0.5281	0.0000	2	8	9	11	0.0177	0.2854	0.6894	0.0076
2	3	8	9	0.0509	0.4630	0.4861	0.0000	2	8	10	11	0.0182	0.2943	0.6807	0.0068
2	3	8	10	0.0500	0.4542	0.4958	0.0000	2	9	10	11	0.0152	0.2657	0.7071	0.0121
2	3	8	11	0.0417	0.4356	0.5227	0.0000	3	4	5	6	0.0500	0.3889	0.5611	0.0000
2	3	9	10	0.0444	0.4481	0.5074	0.0000	3	4	5	7	0.0381	0.3476	0.6143	0.0000
2	3	9	11	0.0404	0.4343	0.5253	0.0000	3	4	5	8	0.0417	0.3583	0.6000	0.0000
2	3	10	11	0.0364	0.4182	0.5455	0.0000	3	4	5	9	0.0370	0.3481	0.6148	0.0000
2	4	5	6	0.0750	0.5000	0.4250	0.0000	3	4	5	10	0.0400	0.3500	0.6100	0.0000
2	4	5	7	0.0571	0.4571	0.4857	0.0000	3	4	5	11	0.0303	0.3242	0.6394	0.0061
2	4	5	8	0.0688	0.4875	0.4438	0.0000	3	4	6	7	0.0397	0.3651	0.5952	0.0000
2	4	5	9	0.0500	0.4444	0.5056	0.0000	3	4	6	8	0.0451	0.3750	0.5799	0.0000
2	4	5	10	0.0650	0.4700	0.4650	0.0000	3	4	6	9	0.0432	0.3735	0.5833	0.0000
2	4	5	11	0.0455	0.4318	0.5227	0.0000	3	4	6	10	0.0361	0.3528	0.6083	0.0028
2	4	6	7	0.0595	0.4762	0.4643	0.0000	3	4	6	11	0.0303	0.3359	0.6263	0.0076
2	4	6	8	0.0781	0.5052	0.4167	0.0000	3	4	7	8	0.0327	0.3333	0.6280	0.0060
2	4	6	9	0.0556	0.4676	0.4769	0.0000	3	4	7	9	0.0291	0.3201	0.6455	0.0053
2	4	6	10	0.0625	0.4792	0.4583	0.0000	3	4	7	10	0.0262	0.3048	0.6595	0.0095
2	4	6	11	0.0455	0.4508	0.5038	0.0000	3	4	7	11	0.0238	0.2944	0.6688	0.0130
2	4	7	8	0.0536	0.4598	0.4866	0.0000	3	4	8	9	0.0301	0.3287	0.6366	0.0046
2	4	7	9	0.0397	0.4087	0.5516	0.0000	3	4	8	10	0.0292	0.3208	0.6438	0.0063
2	4	7	10	0.0429	0.4250	0.5321	0.0000	3	4	8	11	0.0246	0.3068	0.6553	0.0133
2	4	7	11	0.0357	0.3961	0.5682	0.0000	3	4	9	10	0.0241	0.3019	0.6648	0.0093
2	4	8	9	0.0451	0.4479	0.5069	0.0000	3	4	9	11	0.0219	0.2912	0.6734	0.0135
2	4	8	10	0.0531	0.4625	0.4844	0.0000	3	4	10	11	0.0197	0.2758	0.6848	0.0197
2	4	8	11	0.0398	0.4347	0.5256	0.0000	3	5	6	7	0.0317	0.3270	0.6349	0.0063
2	4	9	10	0.0361	0.4083	0.5556	0.0000	3	5	6	8	0.0306	0.3222	0.6389	0.0083
2	4	9	11	0.0303	0.3763	0.5934	0.0000	3	5	6	9	0.0346	0.3383	0.6198	0.0074
2	4	10	11	0.0318	0.3955	0.5727	0.0000	3	5	6	10	0.0311	0.3222	0.6378	0.0089
2	5	6	7	0.0429	0.4048	0.5524	0.0000	3	5	6	11	0.0242	0.3010	0.6606	0.0141
2	5	6	8	0.0458	0.4167	0.5375	0.0000	3	5	7	8	0.0238	0.2833	0.6762	0.0167
2	5	6	9	0.0407	0.3963	0.5630	0.0000	3	5	7	9	0.0243	0.2899	0.6709	0.0148
2	5	6	10	0.0467	0.4167	0.5367	0.0000	3	5	7	10	0.0248	0.2876	0.6724	0.0152
2	5	6	11	0.0333	0.3788	0.5879	0.0000	3	5	7	11	0.0199	0.2649	0.6944	0.0208
2	5	7	8	0.0357	0.3786	0.5857	0.0000	3	5	8	9	0.0222	0.2815	0.6759	0.0204
2	5	7	9	0.0317	0.3587	0.6095	0.0000	3	5	8	10	0.0233	0.2833	0.6750	0.0183
2	5	7	10	0.0371	0.3857	0.5771	0.0000	3	5	8	11	0.0182	0.2561	0.6985	0.0273
2	5	7	11	0.0286	0.3481	0.6234	0.0000	3	5	9	10	0.0222	0.2844	0.6756	0.0178
2	5	8	9	0.0306	0.3583	0.6111	0.0000	3	5	9	11	0.0182	0.2626	0.6949	0.0242
2	5	8	10	0.0375	0.3875	0.5750	0.0000	3	5	10	11	0.0182	0.2618	0.6958	0.0242

Probability Table for p = 4 (cont.)

scale				values of inconsistency				scale				values of inconsistency			
k_1	k_2	k_3	k_4	0	1/3	2/3	1	k_1	k_2	k_3	k_4	0	1/3	2/3	1
3	6	7	8	0.0238	0.2897	0.6726	0.0139	4	7	8	9	0.0139	0.2123	0.7163	0.0575
3	6	7	9	0.0265	0.3069	0.6561	0.0106	4	7	8	10	0.0143	0.2152	0.7125	0.0580
3	6	7	10	0.0206	0.2746	0.6857	0.0190	4	7	8	11	0.0122	0.2013	0.7183	0.0682
3	6	7	11	0.0188	0.2641	0.6926	0.0245	4	7	9	10	0.0111	0.1873	0.7222	0.0794
3	6	8	9	0.0247	0.3009	0.6605	0.0139	4	7	9	11	0.0101	0.1782	0.7237	0.0880
3	6	8	10	0.0208	0.2722	0.6861	0.0208	4	7	10	11	0.0097	0.1766	0.7234	0.0903
3	6	8	11	0.0177	0.2588	0.6944	0.0290	4	8	9	10	0.0118	0.1972	0.7236	0.0674
3	6	9	10	0.0210	0.2852	0.6753	0.0185	4	8	9	11	0.0101	0.1831	0.7279	0.0789
3	6	9	11	0.0191	0.2761	0.6824	0.0224	4	8	10	11	0.0102	0.1858	0.7250	0.0790
3	6	10	11	0.0152	0.2414	0.7071	0.0364	4	9	10	11	0.0081	0.1596	0.7258	0.1066
3	7	8	9	0.0172	0.2474	0.7037	0.0317	5	6	7	8	0.0143	0.2048	0.7119	0.0690
3	7	8	10	0.0155	0.2310	0.7143	0.0393	5	6	7	9	0.0138	0.2011	0.7122	0.0730
3	7	8	11	0.0141	0.2229	0.7186	0.0444	5	6	7	10	0.0143	0.2067	0.7114	0.0676
3	7	9	10	0.0148	0.2328	0.7153	0.0370	5	6	7	11	0.0113	0.1836	0.7195	0.0857
3	7	9	11	0.0139	0.2270	0.7196	0.0394	5	6	8	9	0.0130	0.1963	0.7120	0.0787
3	7	10	11	0.0121	0.2087	0.7273	0.0519	5	6	8	10	0.0142	0.2042	0.7108	0.0708
3	8	9	10	0.0139	0.2250	0.7204	0.0407	5	6	8	11	0.0106	0.1788	0.7205	0.0902
3	8	9	11	0.0126	0.2163	0.7239	0.0471	5	6	9	10	0.0126	0.1963	0.7141	0.0770
3	8	10	11	0.0114	0.2015	0.7295	0.0576	5	6	9	11	0.0101	0.1751	0.7192	0.0956
3	9	10	11	0.0108	0.2007	0.7347	0.0539	5	6	10	11	0.0103	0.1800	0.7212	0.0885
4	5	6	7	0.0238	0.2714	0.6786	0.0262	5	7	8	9	0.0103	0.1730	0.7175	0.0992
4	5	6	8	0.0271	0.2875	0.6646	0.0208	5	7	8	10	0.0114	0.1843	0.7164	0.0879
4	5	6	9	0.0222	0.2630	0.6852	0.0296	5	7	8	11	0.0091	0.1630	0.7208	0.1071
4	5	6	10	0.0250	0.2750	0.6767	0.0233	5	7	9	10	0.0102	0.1746	0.7181	0.0971
4	5	6	11	0.0182	0.2455	0.6985	0.0379	5	7	9	11	0.0084	0.1558	0.7186	0.1172
4	5	7	8	0.0214	0.2607	0.6875	0.0304	5	7	10	11	0.0088	0.1642	0.7221	0.1049
4	5	7	9	0.0175	0.2349	0.7063	0.0413	5	8	9	10	0.0094	0.1678	0.7167	0.1061
4	5	7	10	0.0200	0.2514	0.6929	0.0357	5	8	9	11	0.0076	0.1475	0.7152	0.1298
4	5	7	11	0.0156	0.2247	0.7104	0.0494	5	8	10	11	0.0082	0.1573	0.7200	0.1145
4	5	8	9	0.0181	0.2444	0.7014	0.0361	5	9	10	11	0.0073	0.1475	0.7192	0.1261
4	5	8	10	0.0213	0.2613	0.6875	0.0300	6	7	8	9	0.0099	0.1680	0.7149	0.1071
4	5	8	11	0.0159	0.2318	0.7068	0.0455	6	7	8	10	0.0095	0.1625	0.7137	0.1143
4	5	9	10	0.0167	0.2356	0.7044	0.0433	6	7	8	11	0.0081	0.1515	0.7127	0.1277
4	5	9	11	0.0131	0.2081	0.7192	0.0596	6	7	9	10	0.0085	0.1550	0.7143	0.1222
4	5	10	11	0.0145	0.2236	0.7109	0.0509	6	7	9	11	0.0077	0.1477	0.7109	0.1337
4	6	7	8	0.0208	0.2574	0.6860	0.0357	6	7	10	11	0.0069	0.1390	0.7082	0.1459
4	6	7	9	0.0172	0.2328	0.7037	0.0463	6	8	9	10	0.0083	0.1532	0.7116	0.1269
4	6	7	10	0.0167	0.2298	0.7048	0.0488	6	8	9	11	0.0072	0.1423	0.7109	0.1397
4	6	7	11	0.0141	0.2143	0.7110	0.0606	6	8	10	11	0.0068	0.1375	0.7068	0.1489
4	6	8	9	0.0185	0.2465	0.6968	0.0382	6	9	10	11	0.0061	0.1296	0.7037	0.1606
4	6	8	10	0.0198	0.2479	0 6948	0.0375	7	8	9	10	0.0063	0.1294	0.7008	0.1635
4	6	8	11	0.0152	0.2292	0.7055	0.0502	7	8	9	11	0.0058	0.1230	0.6973	0.1739
4	6	9	10	0.0148	0.2204	0.7102	0.0546	7	8	10	11	0.0055	0.1201	0.6961	0.1782
4	6	9	11	0.0126	0.2045	0.7172	0.0657	7	9	10	11	0.0049	0.1120	0.6883	0.1948
4	6	10	11	0.0121	0.2015	0.7174	0.0689	8	9	10	11	0.0045	0.1066	0.6813	0.2076

Probability Table for p=5

scale					values of inconsistency				
k_1	k_2	k_3	k_4	k_5	**0**	**1/4**	**1/2**	**3/4**	**1**
2	3	4	5	6	0.0306	0.2611	0.6028	0.1056	0.0000
2	3	4	5	7	0.0214	0.2190	0.6214	0.1381	0.0000
2	3	4	5	8	0.0250	0.2354	0.6188	0.1208	0.0000
2	3	4	5	9	0.0204	0.2167	0.6204	0.1426	0.0000
2	3	4	5	10	0.0233	0.2250	0.6200	0.1317	0.0000
2	3	4	5	11	0.0167	0.2000	0.6167	0.1667	0.0000
2	3	4	6	7	0.0238	0.2401	0.6091	0.1270	0.0000
2	3	4	6	8	0.0295	0.2569	0.6042	0.1094	0.0000
2	3	4	6	9	0.0247	0.2423	0.6049	0.1281	0.0000
2	3	4	6	10	0.0236	0.2375	0.6069	0.1319	0.0000
2	3	4	6	11	0.0177	0.2159	0.6061	0.1604	0.0000
2	3	4	7	8	0.0193	0.2143	0.6190	0.1473	0.0000
2	3	4	7	9	0.0159	0.1944	0.6190	0.1706	0.0000
2	3	4	7	10	0.0155	0.1929	0.6179	0.1738	0.0000
2	3	4	7	11	0.0130	0.1775	0.6126	0.1970	0.0000
2	3	4	8	9	0.0174	0.2083	0.6215	0.1528	0.0000
2	3	4	8	10	0.0188	0.2115	0.6177	0.1521	0.0000
2	3	4	8	11	0.0142	0.1932	0.6155	0.1771	0.0000
2	3	4	9	10	0.0139	0.1880	0.6176	0.1806	0.0000
2	3	4	9	11	0.0118	0.1726	0.6103	0.2054	0.0000
2	3	4	10	11	0.0114	0.1712	0.6106	0.2068	0.0000
2	3	5	6	7	0.0175	0.1984	0.6175	0.1667	0.0000
2	3	5	6	8	0.0181	0.2028	0.6139	0.1653	0.0000
2	3	5	6	9	0.0185	0.2037	0.6123	0.1654	0.0000
2	3	5	6	10	0.0178	0.1989	0.6156	0.1678	0.0000
2	3	5	6	11	0.0131	0.1788	0.6091	0.1990	0.0000
2	3	5	7	8	0.0131	0.1690	0.6060	0.2119	0.0000
2	3	5	7	9	0.0127	0.1672	0.6042	0.2159	0.0000
2	3	5	7	10	0.0133	0.1705	0.6038	0.2124	0.0000
2	3	5	7	11	0.0104	0.1515	0.5974	0.2407	0.0000
2	3	5	8	9	0.0120	0.1657	0.6046	0.2176	0.0000
2	3	5	8	10	0.0133	0.1717	0.6042	0.2108	0.0000
2	3	5	8	11	0.0098	0.1500	0.5977	0.2424	0.0000
2	3	5	9	10	0.0119	0.1659	0.6044	0.2178	0.0000
2	3	5	9	11	0.0094	0.1481	0.5960	0.2465	0.0000
2	3	5	10	11	0.0097	0.1515	0.5958	0.2430	0.0000
2	3	6	7	8	0.0139	0.1796	0.6131	0.1935	0.0000
2	3	6	7	9	0.0141	0.1808	0.6111	0.1940	0.0000
2	3	6	7	10	0.0119	0.1675	0.6095	0.2111	0.0000
2	3	6	7	11	0.0101	0.1544	0.6010	0.2345	0.0000
2	3	6	8	9	0.0139	0.1836	0.6096	0.1929	0.0000
2	3	6	8	10	0.0132	0.1736	0.6076	0.2056	0.0000
2	3	6	8	11	0.0101	0.1578	0.6016	0.2304	0.0000
2	3	6	9	10	0.0117	0.1710	0.6062	0.2111	0.0000
2	3	6	9	11	0.0101	0.1588	0.5988	0.2323	0.0000
2	3	6	10	11	0.0086	0.1449	0.5949	0.2515	0.0000
2	3	7	8	9	0.0093	0.1435	0.5939	0.2533	0.0000
2	3	7	8	10	0.0089	0.1393	0.5905	0.2613	0.0000
2	3	7	8	11	0.0076	0.1288	0.5828	0.2808	0.0000
2	3	7	9	10	0.0079	0.1333	0.5868	0.2720	0.0000
2	3	7	9	11	0.0072	0.1265	0.5791	0.2872	0.0000
2	3	7	10	11	0.0065	0.1190	0.5736	0.3009	0.0000
2	3	8	9	10	0.0079	0.1338	0.5898	0.2685	0.0000
2	3	8	9	11	0.0067	0.1233	0.5812	0.2887	0.0000
2	3	8	10	11	0.0064	0.1197	0.5769	0.2970	0.0000
2	3	9	10	11	0.0057	0.1131	0.5721	0.3091	0.0000
2	4	5	6	7	0.0143	0.1750	0.6048	0.2060	0.0000
2	4	5	6	8	0.0177	0.1938	0.6073	0.1813	0.0000
2	4	5	6	9	0.0130	0.1676	0.6028	0.2167	0.0000
2	4	5	6	10	0.0158	0.1817	0.6050	0.1975	0.0000
2	4	5	6	11	0.0106	0.1553	0.5970	0.2371	0.0000
2	4	5	7	8	0.0125	0.1634	0.6000	0.2241	0.0000
2	4	5	7	9	0.0095	0.1397	0.5873	0.2635	0.0000
2	4	5	7	10	0.0114	0.1550	0.5929	0.2407	0.0000
2	4	5	7	11	0.0084	0.1325	0.5805	0.2786	0.0000
2	4	5	8	9	0.0104	0.1514	0.5979	0.2403	0.0000
2	4	5	8	10	0.0131	0.1675	0.6013	0.2181	0.0000
2	4	5	8	11	0.0091	0.1426	0.5920	0.2563	0.0000
2	4	5	9	10	0.0094	0.1433	0.5883	0.2589	0.0000
2	4	5	9	11	0.0071	0.1212	0.5727	0.2990	0.0000
2	4	5	10	11	0.0082	0.1350	0.5823	0.2745	0.0000
2	4	6	7	8	0.0134	0.1711	0.5997	0.2158	0.0000
2	4	6	7	9	0.0099	0.1455	0.5913	0.2533	0.0000
2	4	6	7	10	0.0107	0.1512	0.5917	0.2464	0.0000
2	4	6	7	11	0.0081	0.1331	0.5817	0.2771	0.0000
2	4	6	8	9	0.0116	0.1620	0.5995	0.2269	0.0000
2	4	6	8	10	0.0141	0.1708	0.5969	0.2182	0.0000
2	4	6	8	11	0.0095	0.1501	0.5919	0.2486	0.0000
2	4	6	9	10	0.0093	0.1431	0.5880	0.2597	0.0000
2	4	6	9	11	0.0072	0.1250	0.5766	0.2912	0.0000
2	4	6	10	11	0.0076	0.1307	0.5792	0.2826	0.0000
2	4	7	8	9	0.0079	0.1295	0.5809	0.2817	0.0000
2	4	7	8	10	0.0089	0.1375	0.5826	0.2710	0.0000
2	4	7	8	11	0.0069	0.1218	0.5735	0.2979	0.0000

Probability Table for p=5 (cont.)

scale					values of inconsistency					scale					values of inconsistency				
k_1	k_2	k_3	k_4	k_5	0	1/4	1/2	3/4	1	K_1	k_2	k_3	k_4	k_5	0	1/4	1/2	3/4	1
2	4	7	9	10	0.0063	0.1131	0.5619	0.3187	0.0000	2	6	8	9	10	0.0051	0.0954	0.5296	0.3699	0.0000
2	4	7	9	11	0.0054	0.1025	0.5473	0.3449	0.0000	2	6	8	9	11	0.0040	0.0838	0.5143	0.3979	0.0000
2	4	7	10	11	0.0055	0.1058	0.5539	0.3347	0.0000	2	6	8	10	11	0.0042	0.0856	0.5155	0.3947	0.0000
2	4	8	9	10	0.0073	0.1250	0.5774	0.2903	0.0000	2	6	9	10	11	0.0034	0.0754	0.4968	0.4244	0.0000
2	4	8	9	11	0.0057	0.1095	0.5657	0.3191	0.0000	2	7	8	9	10	0.0036	0.0756	0.4921	0.4288	0.0000
2	4	8	10	11	0.0063	0.1170	0.5699	0.3068	0.0000	2	7	8	9	11	0.0031	0.0689	0.4769	0.4511	0.0000
2	4	9	10	11	0.0045	0.0947	0.5422	0.3586	0.0000	2	7	8	10	11	0.0031	0.0698	0.4818	0.4453	0.0000
2	5	6	7	8	0.0083	0.1256	0.5655	0.3006	0.0000	2	7	9	10	11	0.0026	0.0622	0.4603	0.4749	0.0000
2	5	6	7	9	0.0074	0.1169	0.5561	0.3196	0.0000	2	8	9	10	11	0.0025	0.0615	0.4621	0.4739	0.0000
2	5	6	7	10	0.0081	0.1243	0.5633	0.3043	0.0000	3	4	5	6	7	0.0095	0.1270	0.5365	0.3270	0.0000
2	5	6	7	11	0.0061	0.1061	0.5459	0.3420	0.0000	3	4	5	6	8	0.0104	0.1326	0.5382	0.3188	0.0000
2	5	6	8	9	0.0074	0.1190	0.5588	0.3148	0.0000	3	4	5	6	9	0.0099	0.1296	0.5358	0.3247	0.0000
2	5	6	8	10	0.0088	0.1283	0.5650	0.2979	0.0000	3	4	5	6	10	0.0094	0.1256	0.5339	0.3311	0.0000
2	5	6	8	11	0.0061	0.1080	0.5500	0.3360	0.0000	3	4	5	6	11	0.0071	0.1111	0.5187	0.3631	0.0000
2	5	6	9	10	0.0070	0.1167	0.5567	0.3196	0.0000	3	4	5	7	8	0.0077	0.1119	0.5155	0.3649	0.0000
2	5	6	9	11	0.0054	0.1000	0.5374	0.3572	0.0000	3	4	5	7	9	0.0069	0.1053	0.5090	0.3788	0.0000
2	5	6	10	11	0.0058	0.1064	0.5467	0.3412	0.0000	3	4	5	7	10	0.0071	0.1067	0.5081	0.3781	0.0000
2	5	7	8	9	0.0056	0.0992	0.5325	0.3627	0.0000	3	4	5	7	11	0.0056	0.0944	0.4926	0.4074	0.0000
2	5	7	8	10	0.0064	0.1089	0.5450	0.3396	0.0000	3	4	5	8	9	0.0069	0.1074	0.5116	0.3741	0.0000
2	5	7	8	11	0.0049	0.0929	0.5256	0.3766	0.0000	3	4	5	8	10	0.0075	0.1104	0.5138	0.3683	0.0000
2	5	7	9	10	0.0054	0.0987	0.5324	0.3635	0.0000	3	4	5	8	11	0.0057	0.0970	0.4955	0.4019	0.0000
2	5	7	9	11	0.0043	0.0860	0.5105	0.3991	0.0000	3	4	5	9	10	0.0063	0.1022	0.5048	0.3867	0.0000
2	5	7	10	11	0.0047	0.0922	0.5255	0.3777	0.0000	3	4	5	9	11	0.0051	0.0906	0.4886	0.4158	0.0000
2	5	8	9	10	0.0053	0.0983	0.5314	0.3650	0.0000	3	4	5	10	11	0.0052	0.0918	0.4879	0.4152	0.0000
2	5	8	9	11	0.0040	0.0833	0.5091	0.4035	0.0000	3	4	6	7	8	0.0079	0.1156	0.5218	0.3547	0.0000
2	5	8	10	11	0.0045	0.0916	0.5243	0.3795	0.0000	3	4	6	7	9	0.0075	0.1120	0.5203	0.3602	0.0000
2	5	9	10	11	0.0038	0.0822	0.5091	0.4048	0.0000	3	4	6	7	10	0.0063	0.1020	0.5075	0.3841	0.0000
2	6	7	8	9	0.0056	0.1005	0.5377	0.3562	0.0000	3	4	6	7	11	0.0054	0.0945	0.4964	0.4037	0.0000
2	6	7	8	10	0.0060	0.1024	0.5384	0.3533	0.0000	3	4	6	8	9	0.0077	0.1157	0.5216	0.3549	0.0000
2	6	7	8	11	0.0046	0.0904	0.5238	0.3812	0.0000	3	4	6	8	10	0.0073	0.1090	0.5122	0.3715	0.0000
2	6	7	9	10	0.0048	0.0915	0.5241	0.3796	0.0000	3	4	6	8	11	0.0057	0.0991	0.5003	0.3949	0.0000
2	6	7	9	11	0.0041	0.0835	0.5079	0.4045	0.0000	3	4	6	9	10	0.0062	0.1031	0.5074	0.3833	0.0000
2	6	7	10	11	0.0039	0.0818	0.5082	0.4061	0.0000	3	4	6	9	11	0.0053	0.0957	0.4969	0.4021	0.0000

Probability Table for p=5 (cont.)

scale					values of inconsistency				
k_1	k_2	k_3	k_4	k_5	0	1/4	1/2	3/4	1
3	4	6	10	11	0.0045	0.0864	0.4818	0.4273	0.0000
3	4	7	8	9	0.0053	0.0916	0.4894	0.4137	0.0000
3	4	7	8	10	0.0051	0.0881	0.4813	0.4256	0.0000
3	4	7	8	11	0.0043	0.0817	0.4716	0.4424	0.0000
3	4	7	9	10	0.0042	0.0802	0.4698	0.4458	0.0000
3	4	7	9	11	0.0038	0.0760	0.4615	0.4586	0.0000
3	4	7	10	11	0.0035	0.0710	0.4487	0.4768	0.0000
3	4	8	9	10	0.0044	0.0831	0.4748	0.4377	0.0000
3	4	8	9	11	0.0038	0.0768	0.4642	0.4552	0.0000
3	4	8	10	11	0.0036	0.0739	0.4561	0.4665	0.0000
3	4	9	10	11	0.0030	0.0665	0.4419	0.4884	0.0002
3	5	6	7	8	0.0056	0.0913	0.4845	0.4187	0.0000
3	5	6	7	9	0.0060	0.0959	0.4917	0.4063	0.0000
3	5	6	7	10	0.0054	0.0902	0.4819	0.4225	0.0000
3	5	6	7	11	0.0043	0.0802	0.4670	0.4485	0.0000
3	5	6	8	9	0.0056	0.0926	0.4864	0.4154	0.0000
3	5	6	8	10	0.0053	0.0886	0.4786	0.4275	0.0000
3	5	6	8	11	0.0040	0.0773	0.4616	0.4571	0.0000
3	5	6	9	10	0.0052	0.0904	0.4835	0.4210	0.0000
3	5	6	9	11	0.0043	0.0813	0.4698	0.4447	0.0000
3	5	6	10	11	0.0038	0.0762	0.4582	0.4618	0.0000
3	5	7	8	9	0.0040	0.0743	0.4495	0.4722	0.0000
3	5	7	8	10	0.0040	0.0748	0.4474	0.4738	0.0000
3	5	7	8	11	0.0032	0.0658	0.4290	0.5019	0.0000
3	5	7	9	10	0.0038	0.0734	0.4478	0.4749	0.0000
3	5	7	9	11	0.0032	0.0658	0.4323	0.4987	0.0000
3	5	7	10	11	0.0031	0.0653	0.4281	0.5036	0.0000
3	5	8	9	10	0.0035	0.0702	0.4417	0.4844	0.0002
3	5	8	9	11	0.0029	0.0618	0.4234	0.5113	0.0007
3	5	8	10	11	0.0029	0.0621	0.4217	0.5129	0.0005
3	5	9	10	11	0.0027	0.0606	0.4215	0.5146	0.0005
3	6	7	8	9	0.0042	0.0776	0.4599	0.4583	0.0000
3	6	7	8	10	0.0036	0.0692	0.4399	0.4871	0.0002

Scale					values of inconsistency				
k_1	k_2	k_3	k_4	k_5	0	1/4	1/2	3/4	1
3	6	7	8	11	0.0031	0.0642	0.4289	0.5029	0.0009
3	6	7	9	10	0.0035	0.0707	0.4471	0.4785	0.0002
3	6	7	9	11	0.0032	0.0672	0.4388	0.4903	0.0005
3	6	7	10	11	0.0026	0.0582	0.4140	0.5238	0.0014
3	6	8	9	10	0.0034	0.0690	0.4421	0.4852	0.0003
3	6	8	9	11	0.0029	0.0640	0.4320	0.5001	0.0010
3	6	8	10	11	0.0025	0.0569	0.4092	0.5295	0.0018
3	6	9	10	11	0.0025	0.0576	0.4167	0.5217	0.0016
3	7	8	9	10	0.0024	0.0536	0.3958	0.5462	0.0021
3	7	8	9	11	0.0022	0.0507	0.3879	0.5565	0.0026
3	7	8	10	11	0.0019	0.0470	0.3723	0.5752	0.0036
3	7	9	10	11	0.0018	0.0457	0.3727	0.5763	0.0035
3	8	9	10	11	0.0017	0.0432	0.3634	0.5872	0.0045
4	5	6	7	8	0.0048	0.0795	0.4452	0.4705	0.0000
4	5	6	7	9	0.0040	0.0709	0.4267	0.4981	0.0003
4	5	6	7	10	0.0043	0.0745	0.4336	0.4874	0.0002
4	5	6	7	11	0.0032	0.0636	0.4115	0.5206	0.0011
4	5	6	8	9	0.0042	0.0741	0.4354	0.4863	0.0000
4	5	6	8	10	0.0048	0.0790	0.4431	0.4731	0.0000
4	5	6	8	11	0.0034	0.0669	0.4197	0.5093	0.0008
4	5	6	9	10	0.0037	0.0693	0.4231	0.5033	0.0006
4	5	6	9	11	0.0029	0.0593	0.4003	0.5359	0.0017
4	5	6	10	11	0.0030	0.0623	0.4073	0.5262	0.0012
4	5	7	8	9	0.0032	0.0623	0.4079	0.5256	0.0010
4	5	7	8	10	0.0036	0.0671	0.4189	0.5095	0.0009
4	5	7	8	11	0.0028	0.0578	0.3959	0.5416	0.0019
4	5	7	9	10	0.0029	0.0584	0.3965	0.5402	0.0021
4	5	7	9	11	0.0023	0.0509	0.3740	0.5693	0.0035
4	5	7	10	11	0.0025	0.0540	0.3848	0.5558	0.0029
4	5	8	9	10	0.0029	0.0600	0.4026	0.5329	0.0015
4	5	8	9	11	0.0023	0.0513	0.3784	0.5648	0.0033
4	5	8	10	11	0.0025	0.0552	0.3898	0.5499	0.0026
4	5	9	10	11	0.0020	0.0477	0.3670	0.5786	0.0047

Probability Table for p=5 (cont.)

scale					values of inconsistency				
k_1	k_2	k_3	k_4	k_5	**0**	**1/4**	**1/2**	**3/4**	**1**
4	6	7	8	9	0.0031	0.0618	0.4044	0.5288	0.0018
4	6	7	8	10	0.0033	0.0622	0.4027	0.5298	0.0021
4	6	7	8	11	0.0026	0.0552	0.3883	0.5503	0.0037
4	6	7	9	10	0.0025	0.0534	0.3796	0.5608	0.0036
4	6	7	9	11	0.0022	0.0488	0.3664	0.5772	0.0054
4	6	7	10	11	0.0021	0.0474	0.3616	0.5828	0.0062
4	6	8	9	10	0.0028	0.0572	0.3906	0.5468	0.0027
4	6	8	9	11	0.0022	0.0505	0.3752	0.5678	0.0043
4	6	8	10	11	0.0023	0.0509	0.3738	0.5685	0.0045
4	6	9	10	11	0.0018	0.0433	0.3486	0.5985	0.0079
4	7	8	9	10	0.0020	0.0457	0.3561	0.5897	0.0065
4	7	8	9	11	0.0017	0.0418	0.3435	0.6044	0.0085
4	7	8	10	11	0.0017	0.0419	0.3438	0.6037	0.0088
4	7	9	10	11	0.0014	0.0358	0.3169	0.6325	0.0135
4	8	9	10	11	0.0014	0.0366	0.3238	0.6265	0.0118
5	6	7	8	9	0.0022	0 0479	0.3536	0.5886	0.0077
5	6	7	8	10	0.0024	0 0496	0.3589	0.5819	0.0071
5	6	7	8	11	0.0018	0.0424	0.3361	0.6100	0.0096
5	6	7	9	10	0.0021	0.0466	0.3510	0.5922	0.0081
5	6	7	9	11	0.0017	0.0409	0.3304	0.6159	0.0112
5	6	7	10	11	0.0017	0.0413	0.3342	0.6125	0.0103
5	6	8	9	10	0.0020	0.0455	0.3458	0.5973	0.0094
5	6	8	9	11	0.0016	0.0389	0.3235	0.6233	0.0127
5	6	8	10	11	0.0017	0.0402	0.3289	0.6178	0.0114
5	6	9	10	11	0.0015	0.0375	0.3202	0.6273	0.0135
5	7	8	9	10	0.0016	0.0383	0.3194	0.6263	0.0144
5	7	8	9	11	0.0013	0.0334	0.2984	0.6484	0.0185
5	7	8	10	11	0.0014	0.0349	0.3075	0.6400	0.0162
5	7	9	10	11	0.0012	0.0323	0.2964	0.6507	0.0193
5	8	9	10	11	0.0011	0.0304	0.2873	0.6583	0.0229
6	7	8	9	10	0.0014	0.0344	0.2993	0.6449	0.0200
6	7	8	9	11	0.0012	0.0315	0.2882	0.6555	0.0236
6	7	8	10	11	0.0011	0.0302	0.2809	0.6618	0.0260
6	7	9	10	11	0.0010	0.0280	0.2720	0.6694	0.0295
6	8	9	10	11	0.0010	0.0272	0.2672	0.6730	0.0317
7	8	9	10	11	0.0008	0.0226	0.2398	0.6922	0.0447

B A C Programming Routine for Computation of the Inconsistency measure

As an input, one must provide the following information in sequence:

- Name of the file in which the output is to be written.
- Name of the file containing the rating or response (each row must correspond to a separate respondent, and the column corresponding to the scale).
- Name of the file containing the scale factor (this text file must contain a row giving the various k_i values).

As an output, among other things, comes the column vector of inconsistency or fuzziness measure corresponding to the respondents, as well as overall measure of inconsistency (aggregating all respondents, treating them as a homogeneous group). The results would appear on the screen as well. Note that this program runs for one attribute at a time and hence when several attributes are repeated, a re-run of the program would be necessary.

```
#include <stdio.h>
#include <math.h>
#define D double
int n_k, n_resp;

int k[25];  /* scale */
int R[100][25]; /* response matrix: column for scale, row for respondent */
D mean[100];
D fuzz[100];
D fuzz_all;
D f[100][20000]; /* change this according to the product of the scales) */
D f_all[20000];
D mean_all ;
int grid ;
char outfilename[20];

void read_input(void);
void step1(void);
void step2(void);
void step3(void);
void output(void);

void main(void)
{      n_k = 4;
       n_resp = 11;
       read_input();
       step1();
       printf("Step 1 over\n");
       step2();
       printf("Step 2 over\n");
       step3();
       printf("Step 3 over\n");
       output();
       return;
}

/* Read k(vector) and R(matrix) here */
void read_input(void)
{      int i, j;
       char filename[20];
       FILE *fp;
       printf("Name of outfilename : ");
       scanf("%s",outfilename);
       printf("Name of file giving ratings : ");
       scanf("%s",filename);
       fp = fopen(filename,"r");
       if (fp == NULL)
       {      printf("CHAOS : Cannot open %s for reading \n",filename);
              exit (0);
       }
       for (i = 1; i <= n_resp; i++)
       {      for (j = 1; j <= n_k; j++)
              {            fscanf(fp,"%d",&R[i][j]);     }
       }
       fclose(fp);
```

```
        printf("Name of file for scale vector : ");
        scanf("%s",filename);
        fp = fopen(filename,"r");
        if (fp == NULL)
        {
              printf("CHAOS : Cannot open %s for reading \n",filename);
              exit (0);
        }
        for (j = 1; j <= n_k; j++)
        {
              fscanf(fp, "%d", &k[j]);
        }
        fclose(fp);
        return;
}
/* end read */

void step1(void)
{     int i, j;
grid = 1;
for (j=1; j <= n_k; j++)
grid *= k[j],
grid +=1;
printf("grid %d\n", grid);

fuzz_all = 0.0;
for ( i = 1; i <= n_resp; i++)
        {
        fuzz[i] = 0.0;
        for (j=1; j <= grid; j++)
              {
              f[i][j] = 0.0;
              f_all[j] = 0.0;
              }
        }
        return;
}

void step2(void)
{     int i, j, lb, ub, s;
for ( i = 1; i <= n_resp; i++)
        {     for (j = 1; j <= n_k; j++)
              {
              lb = grid / k[j];
              ub = lb * R[i][j];
              lb *= (R[i][j] -1);
              lb += 1;
              ub += 1;
                    for (s = lb; s <= ub; s++)
                          {
                          f[i][s] += 1.0;
                          f_all[s] += 1.0;
                          }
              }
        }
        return;
}
```

```
void step3(void)
{     int i, j, ind;
for (j=1; j <= grid; j++)
      {     if (f_all[j] > fuzz_all)
            {
            fuzz_all = f_all[j];
            mean_all = (j-1) * 1.0 / (grid -1);
            }
      }
for ( i = 1; i <= n_resp; i++)
      {
      for (j=1; j <= grid; j++)
            {
            if (f[i][j] > fuzz[i])
                  {
                  fuzz[i] = f[i][j];
                  mean[i] = (j-1) * 1.0 / (grid -1) ;
                  }
            }
      }
            ind = 0;
for (j=grid; j >= 1; j--)
      {
       if ( (f_all[j] >= fuzz_all) & (ind == 0) )
            {
            ind = 1;
            mean_all += (j-1) * 1.0 / (grid -1);
            mean_all  *= 0.5;
            }
      }
for ( i = 1; i <= n_resp; i++)
{           ind = 0;
for (j=grid; j >= 1; j--)
      {
      if ((f[i][j] >= fuzz[i]) & (ind == 0) )
            {
            ind = 1;
            mean[i] += (j-1) * 1.0 / (grid -1);
            mean[i]  *= 0.5;
            }
      }
}

            fuzz_all /= ( (-1.0) * n_k * n_resp);
            fuzz_all += 1.0;
            fuzz_all *= (n_k)/(n_k-1.0);
for ( i = 1; i <= n_resp; i++)
      {
            fuzz[i] /= (-1.0) * n_k ;
            fuzz[i] += 1.0;
            fuzz[i] *= n_k;
            fuzz[i] /= (n_k - 1.0);
      }
      return;
```

```
/* print commands and output here */
void output(void)
{
int i, j;
FILE *outf;

            printf("Overall fuzziness level %lf", fuzz_all);
            printf("Overall rating (out of 1) %lf", mean_all);
            printf("\n respondent-wise fuzziness\n");
            for (i = 1; i <= n_resp; i++)
                   printf("%lf\n", fuzz[i]);
            printf("\n respondent-wise mean rating\n");
            for (i = 1; i <= n_resp; i++)
                   printf("%lf\n", mean[i]);

            outf = fopen(outfilename,"a");
            fprintf(outf, "\nThe scale vector\n");
            for (j = 1; j <= n_k; j++)
                  fprintf(outf, "%2d ",k[j]);
            fprintf(outf, "\n");
            fprintf(outf, "\nThe Rating matrix \n");
            for (i = 1; i <= n_resp; i++)
            {
                  for (j = 1; j <= n_k; j++)
                        fprintf(outf, "%2d",R[i][j]);
                  fprintf(outf, "\n");
            }
            fprintf(outf, "\n");
            fprintf(outf, "\nOverall fuzziness level\n");
              fprintf(outf,"%lf ",fuzz_all);
            fprintf(outf, "\n");
            fprintf(outf, "\nOverall Mean rating\n");
              fprintf(outf,"%lf ",mean_all);
            fprintf(outf, "\n");
            fprintf(outf, "\n respondent-wise fuzziness\n");
            for (i = 1; i <= n_resp; i++)
            {
                        fprintf(outf, "%lf",fuzz[i]);
                  fprintf(outf, "\n");
            }
            fprintf(outf, "\n respondent-wise mean rating\n");
            for (i = 1; i <= n_resp; i++)
            {
                        fprintf(outf, "%lf",mean[i]);
                  fprintf(outf, "\n");
            }

            fclose(outf);

            return;
```

C Summary of Results containing Inconsistency matrix and uni-dimensional Inference

C.1 Tables and Results from Cricket Survey Data

TABLE - 1

Fuzziness

Respondent No.	FIELDING				SINCERITY				Ability to face adverse situation				BATTING				BOWLING				CAPTAINCY			
	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK
1	0	0.75	0.5	0.5	0	0	0.5	0.5	0	0.75	0.5	0.5	0.5	0	0.5	0.5	0.5	0.5	0.75	0	0	0	0	0.5
2	0.25	0	0.25	0.25	0.5	0	0.25	0.25	0	0.25	0.25	0	0.5	0	0.25	0.25	0	0.25	0.25	0.25	0.25	0	0.25	0.25
3	0	0.25	0.25	0.25	0.25	0	0	0.25	0	0	0.25	0.25	0.25	0	0	0.25	0.25	0.25	0.5	0.5	0.25	0.25	0	0.25
4	0.25	0.25	0.25	0.25	0.25	0	0	0.25	0	0	0.5	0.75	0.25	0	0.25	0.25	0	0.25	0	0	0.5	0	0.5	0
5	0.25	0.25	0.5	0	0.25	0	0.25	0.5	0	0.25	0	0.5	0.5	0	0	0.5	0.25	0.5	0.75	0.5	0.5	0.5	0	0.5
6	0	0.25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.25	0	0.5	0
7	0	0.5	0	0	0.25	0	0.25	0.5	0.25	0.25	0.25	0.5	0.25	0	0.5	0.25	0	0	0	0	0.25	0	0	0
8	0	0.25	0	0.25	0	0	0.5	0.25	0	0.25	0	0.25	0	0.25	0.5	0	0	0	0.5	0	0	0	0	0
9	0	0.25	0.5	0.25	0	0	0.25	0.5	0.5	0	0.5	0.25	0.5	0	0	0.25	0.5	0.25	0.25	0	0.25	0	0.5	0.25
10	0	0.5	0.25	0.25	0	0.25	0.5	0	0	0.25	0.25	0.25	0.25	0	0	0.5	0.5	0.25	0.25	0	0	0	0.25	0.25
11	0	0.25	0.25	0.25	0	0	0	0	0	0.75	0.25	0.25	0	0.25	0	0.5	0	0.5	0.5	0.25	0	0.25	0.25	0.5
12	0	0.25	0	0	0.25	0	0	0	0	0	0	0	0	0	0	0.25	0.25	0.25	0	0	0	0	0	0
13	0	0	0.5	0.5	0.25	0.25	0.25	0.5	0	0.5	0.25	0.25	0.25	0.25	0.25	0	0	0	0.25	0	0.5	0.25	0.5	0.25
14	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.5	0.25	0.5	0	0.5	0.5	0.25	0.25	0	0.25	0.25	0.25	0.5	0.5
15	0	0.5	0.5	0.5	0.25	0	0.25	0.5	0	0.5	0.25	0.5	0	0.25	0.5	0.5	0.5	0.25	0.5	0.25	0.25	0	0.5	0.25
16	0	0	0	0.25	0.25	0	0.25	0.5	0	0	0.25	0	0	0	0.25	0.25	0	0	0	0.25	0	0.25	0.5	0.5
17	0	0	0.25	0	0.25	0.5	0.25	0	0.5	0	0.25	0.25	0	0	0	0.25	0	0	0.25	0.25	0	0	0	0
18	0	0	0.5	0.5	0.25	0.25	0.5	0.5	0	0.25	0.25	0.5	0	0	0	0.25	0	0.5	0	0	0.25	0.25	0.25	0.25
19	0	0.25	0.25	0	0	0	0	0	0	0	0	0.25	0.25	0.5	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0	0
20	0	0.25	0	0.25	0	0.25	0	0.25	0	0.25	0	0.25	0.25	0	0.25	0	0	0.25	0	0	0.25	0	0.25	0.25
21	0	0	0.25	0.25	0.25	0	0	0.25	0	0	0.25	0.25	0.25	0	0.25	0	0.25	0.25	0	0.5	0.25	0.25	0.5	0.5
22	0	0.25	0.25	0.5	0.5	0.25	0	0.25	0.5	0.5	0.75	0.5	0.25	0.25	0.5	0.75	0.5	0.75	0	0.5	0.25	0.25	0.25	0.5
23	0	0.25	0.5	0.5	0	0.25	0	0.25	0	0.5	0.25	0.25	0	0	0	0.5	0.25	0.25	0.25	0	0.5	0	0.25	0.5
24	0	0	0.25	0	0.5	0	0.25	0	0.5	0.25	0	0.5	0.25	0	0.25	0	0.25	0.25	0	0	0	0	0.25	0.25
25	0	0.25	0.5	0.5	0	0	0	0.25	0	0.25	0.25	0.25	0.25	0	0.25	0	0.25	0.25	0.25	0.25	0.5	0.5	0.5	0
26	0	0.25	0	0	0.25	0	0.25	0.25	0.25	0.25	0.25	0	0.25	0.25	0.5	0	0.5	0.25	0.25	0.25	0.5	0.25	0.25	0.5
27	0.25	0	0.25	0.25	0	0.25	0.25	0	0	0.75	0.5	0.25	0.25	0.5	0.5	0	0.25	0.25	0.25	0	0.25	0.5	0.25	0.25
28	0	0.25	0.5	0.25	0.25	0	0	0.75	0	0.5	0.25	0.25	0.25	0	0	0.5	0.25	0.25	0.25	0	0	0	0.25	0.25
29	0	0	0.25	0.5	0.25	0	0	0.25	0	0.25	0	0.5	0.25	0	0.25	0	0.5	0	0	0.25	0.25	0.25	0.5	0.25
30	0	0.25	0	0	0.5	0	0	0.25	0.25	0.25	0	0.5	0.25	0	0.5	0.25	0	0.25	0.25	0.25	0	0	0	0
31	0.25	0.25	0.5	0.5	0	0.25	0.75	0.25	0	0.5	0.25	0.5	0.25	0	0.25	0.25	0.5	0.25	0.5	0.75	0	0.25	0	0.25

TABLE - 1 (cont.)

Fuzziness

Respondent No	FIELDING				SINCERITY				Ability to face adverse situation				BATTING				BOWLING				CAPTAINCY			
	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK
32	0	0.25	0.5	0.25	0.5	0.25	0.25	0.25	0.5	0.25	0.5	0.5	0	0	0.25	0.5	0.25	0.25	0	0	0	0.25	0.5	0.25
33	0	0.25	0.25	0.5	0.25	0.25	0	0.25	0	0.25	0	0.25	0.25	0	0	0	0	0	0	0	0.25	0	0.25	0.25
34	0	0.25	0	0.25	0	0.25	0.25	0	0	0.5	0.25	0.5	0	0	0	0.25	0	0.5	0	0.25	0.25	0.5	0.25	0.25
35	0.25	0.25	0.25	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0.25	0	0	0	0	0.25	0
36	0	0	0.25	0	0	0.25	0.25	0.25	0	0.25	0.25	0.5	0.25	0	0	0.25	0.25	0.25	0.25	0	0	0	0	0
37	0	0.5	0.25	0.25	0.25	0.25	0.5	0.25	0.5	0.5	0.25	0.25	0.25	0	0.25	0	0	0.5	0.25	0	0	0.25	0.25	0.25
38	0	0.5	0.25	0.25	0.5	0.5	0.5	0.25	0.25	0.5	0.5	0.25	0.25	0	0.25	0.25	0	0.5	0.25	0	0	0.25	0	0.25
39	0.5	0.25	0	0	0	0.25	0.25	0	0.25	0.25	0	0.25	0.25	0	0	0	0.25	0.25	0	0	0.25	0.25	0.25	0
40	0.25	0.25	0.5	0	0.5	0	0.5	0.25	0.5	0.75	0.25	0.25	0.25	0	0.25	0	0	0	0	0	0	0	0.25	0.25
41	0	0.75	0.5	0.25	0.25	0.25	0.25	0.5	0	0.5	0.25	0.75	0.5	0.5	0.25	0.5	0.5	0.5	0.25	0.5	0	0.25	0.25	0.5
42	0.25	0	0	0.25	0.5	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.5	0	0.25	0	0	0.25	0.5
43	0.25	0.25	0.5	0	0.25	0	0	0	0	0	0.25	0.25	0.25	0	0.25	0.25	0.5	0.25	0.25	0.5	0.25	0	0.5	0
44	0.25	0	0	0	0	0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0	0	0	0	0.25	0	0	0.25	0.25	0.5	0.5
45	0.25	0.25	0.25	0.25	0.5	0.5	0.25	0	0	0.5	0.25	0.5	0	0	0	0.25	0	0.5	0.5	0.5	0.25	0.25	0.25	0.5
46	0	0.25	0	0.25	0.5	0.5	0.5	0.25	0	0	0.5	0	0	0	0.25	0	0	0.25	0	0.25	0.5	0.5	0.5	0.25
47	0.25	0	0	0	0	0	0	0	0.25	0.25	0.25	0.25	0.5	0.25	0.5	0.25	0.5	0.5	0	0.25	0.25	0	0.25	0.25
48	0	0.25	0	0	0.25	0	0	0	0	0.25	0.25	0.25	0.25	0	0	0.25	0	0.5	0	0	0	0	0.25	0.25
49	0	0.25	0	0.25	0	0	0	0	0	0	0	0	0.25	0	0.25	0.25	0	0	0	0	0.5	0.5	0.25	0.25
50	0	0.25	0.5	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.5	0.5	0	0	0.25	0.5	0.25	0	0.5	0	0.25	0.5	0.5	0.25
51	0	0.25	0	0.25	0	0	0.25	0	0	0.25	0.25	0.25	0	0	0.25	0	0.25	0.25	0.25	0.25	0	0.5	0.5	0
52	0	0.25	0.25	0	0	0.25	0.25	0	0.25	0	0	0	0	0	0	0	0	0.25	0	0.25	0.25	0.5	0	0.25
53	0	0	0	0	0	0.25	0	0.25	0	0	0	0	0	0	0.25	0.25	0	0.5	0	0	0	0	0.25	0
54	0	0	0	0.25	0.25	0.5	0	0	0.25	0.25	0	0.5	0.25	0.25	0.5	0	0	0.25	0.25	0	0.25	0	0.25	0.25
55	0	0	0	0.25	0	0	0.25	0.5	0.25	0	0.25	0.25	0.5	0	0.25	0.5	0.25	0.25	0.25	0	0.25	0.5	0.5	0.25
56	0	0.25	0.5	0.25	0.25	0.5	0.25	0.25	0	0	0	0	0	0	0	0.25	0.25	0	0.25	0.25	0	0	0.5	0.75
57	0	0.25	0.5	0.25	0.5	0	0.25	0.25	0.25	0	0	0.5	0.25	0	0.25	0	0	0	0.25	0.25	0.25	0	0.25	0.25
58	0.25	0.25	0.25	0.5	0.25	0	0.25	0.5	0	0	0.5	0.25	0	0	0.25	0.25	0	0.25	0.25	0.25	0.25	0.25	0.25	0.5
comb.	0.38	0.47	0.74	0.66	0.74	0.61	0.69	0.69	0.59	0.71	0.71	0.76	0.55	0.3	0.61	0.56	0.66	0.68	0.59	0.66	0.72	0.7	0.76	0.77
mean	0.07	0.22	0.24	0.22	0.2	0.14	0.19	0.22	0.12	0.25	0.23	0.3	0.2	0.07	0.21	0.22	0.18	0.26	0.19	0.16	0.18	0.17	0.27	0.25

TABLE - 2

Mode Rating on (0,1) scale

Respondent No.	FIELDING				SINCERITY				Ability to face adverse situation				BATTING				BOWLING				CAPTAINCY			
	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK
1	0.955	0.675	0.1	0.31	0.955	0.955	0.258	0.573	0.955	0.586	0.258	0.31	0.573	0.955	0.722	0.136	0.427	0.551	0.278	0.955	0.955	0.955	0.05	0.357
2	0.722	0.563	0.254	0.156	0.31	0.955	0.667	0.667	0.667	0.789	0.333	0.127	0.5	0.955	0.578	0.127	0.127	0.414	0.254	0.864	0.722	0.955	0.31	0.211
3	0.955	0.667	0.586	0.619	0.667	0.955	0.955	0.789	0.955	0.955	0.955	0.753	0.667	0.955	0.955	0.5	0.551	0.563	0.7	0.817	0.487	0.667	0.19	0.619
4	0.746	0.829	0.652	0.652	0.667	0.955	0.838	0.69	0.753	0.873	0.573	0.556	0.746	0.955	0.817	0.045	0.162	0.449	0.162	0.873	0.578	0.955	0.61	0.247
5	0.955	0.789	0.578	0.247	0.667	0.955	0.838	0.5	0.955	0.789	0.955	0.258	0.817	0.955	0.955	0.258	0.667	0.633	0.8	0.633	0.789	0.619	0.95	0.258
6	0.899	0.586	0.127	0.382	0.955	0.955	0.955	0.955	0.955	0.955	0.955	0.955	0.551	0.955	0.955	0.279	0.551	0.437	0.652	0.873	0.633	0.955	0.38	0.838
7	0.955	0.619	0.191	0.348	0.586	0.955	0.333	0.404	0.944	0.929	0.333	0.214	0.667	0.955	0.5	0.136	0.045	0.414	0.5	0.789	0.667	0.955	0.33	0.162
8	0.955	0.722	0.101	0.414	0.955	0.955	0.381	0.5	0.955	0.667	0.162	0.367	0.955	0.773	0.522	0.162	0.279	0.191	0.487	0.721	0.789	0.809	0.13	0.414
9	0.873	0.667	0.136	0.513	0.899	0.899	0.303	0.573	0.633	0.618	0.278	0.773	0.722	0.955	0.955	0.279	0.247	0.278	0.513	0.789	0.789	0.899	0.17	0.652
10	0.899	0.718	0.586	0.586	0.899	0.899	0.586	0.789	0.955	0.809	0.667	0.667	0.809	0.955	0.955	0.5	0.5	0.573	0.586	0.955	0.838	0.955	0.56	0.667
11	0.809	0.563	0.563	0.563	0.838	0.838	0.838	0.838	0.838	0.578	0.667	0.667	0.753	0.753	0.753	0.333	0.586	0.31	0.31	0.789	0.789	0.667	0.69	0.533
12	0.721	0.618	0.618	0.618	0.633	0.618	0.618	0.618	0.69	0.563	0.69	0.586	0.586	0.753	0.753	0.414	0.382	0.5	0.5	0.753	0.5	0.69	0.5	0.5
13	0.789	0.563	0.238	0.31	0.513	0.513	0.513	0.606	0.667	0.343	0.513	0.633	0.513	0.817	0.513	0.211	0.191	0.191	0.367	0.586	0.487	0.633	0.49	0.551
14	0.817	0.586	0.333	0.333	0.817	0.817	0.586	0.619	0.789	0.753	0.5	0.333	0.619	0.899	0.619	0.171	0.367	0.367	0.551	0.619	0.5	0.633	0.31	0.333
15	0.101	0.303	0.764	0.586	0.101	0.045	0.367	0.409	0.045	0.5	0.5	0.437	0.162	0.31	0.191	0.5	0.414	0.381	0.485	0.5	0.191	0.045	0.72	0.5
16	0.899	0.721	0.449	0.382	0.773	0.899	0.633	0.667	0.838	0.667	0.551	0.563	0.69	0.955	0.722	0.31	0.333	0.449	0.618	0.844	0.753	0.619	0.49	0.333
17	0.899	0.667	0.5	0.333	0.667	0.817	0.667	0.838	0.586	0.667	0.667	0.667	0.652	0.955	0.899	0.333	0.5	0.333	0.667	0.817	0.5	0.5	0.5	0.5
18	0.838	0.69	0.422	0.422	0.333	0.69	0.657	0.657	0.69	0.5	0.5	0.439	0.586	0.789	0.789	0.183	0.101	0.389	0.652	0.652	0.227	0.5	0.23	0.279
19	0.414	0.437	0.381	0.414	0.449	0.449	0.449	0.449	0.348	0.449	0.333	0.381	0.333	0.69	0.333	0.278	0.31	0.333	0.31	0.437	0.333	0.449	0.33	0.333
20	0.838	0.722	0.348	0.449	0.753	0.817	0.753	0.722	0.838	0.667	0.667	0.817	0.753	0.955	0.753	0.211	0.31	0.437	0.5	0.789	0.789	0.899	0.51	0.513
21	0.838	0.652	0.227	0.633	0.873	0.873	0.873	0.873	0.753	0.955	0.618	0.573	0.722	0.955	0.817	0.279	0.333	0.333	0.586	0.619	0.817	0.838	0.31	0.578
22	0.333	0.586	0.513	0.667	0.367	0.757	0.586	0.5	0.367	0.633	0.53	0.578	0.414	0.854	0.733	0.656	0.333	0.595	0.667	0.833	0.333	0.817	0.62	0.551
23	0.721	0.563	0.439	0.444	0.348	0.722	0.551	0.838	0.333	0.643	0.5	0.764	0.551	0.899	0.667	0.439	0.31	0.5	0.5	0.838	0.303	0.753	0.37	0.563
24	0.873	0.667	0.5	0.333	0.333	0.955	0.667	0.955	0.487	0.573	0.551	0.556	0.551	0.955	0.753	0.045	0.31	0.367	0.551	0.955	0.191	0.955	0.59	0.817
25	0.955	0.753	0.343	0.343	0.789	0.789	0.789	0.381	0.955	0.171	0.667	0.171	0.789	0.955	0.667	0.162	0.513	0.513	0.513	0.513	0.7	0.7	0.75	0.31
26	0.838	0.667	0.191	0.348	0.697	0.789	0.254	0.414	0.667	0.817	0.367	0.437	0.667	0.817	0.478	0.247	0.5	0.619	0.586	0.633	0.591	0.753	0.17	0.214
27	0.789	0.586	0.227	0.619	0.955	0.722	0.69	0.955	0.899	0.386	0.697	0.789	0.618	0.955	0.833	0.211	0.5	0.367	0.667	0.955	0.619	0.5	0.28	0.798
28	0.838	0.619	0.333	0.5	0.586	0.899	0.899	0.629	0.838	0.611	0.573	0.573	0.667	0.955	0.955	0.31	0.367	0.563	0.753	0.955	0.667	0.955	0.62	0.5
29	0.899	0.721	0.414	0.303	0.773	0.753	0.753	0.773	0.873	0.619	0.618	0.357	0.697	0.873	0.746	0.247	0.449	0.449	0.449	0.333	0.817	0.619	0.31	0.303
30	0.955	0.667	0.045	0.045	0.556	0.838	0.838	0.586	0.817	0.817	0.789	0.31	0.667	0.838	0.817	0.5	0.045	0.045	0.513	0.513	0.69	0.789	0.21	0.247
31	0.789	0.513	0.238	0.238	0.69	0.551	0.421	0.367	0.69	0.667	0.5	0.394	0.586	0.789	0.667	0.162	0.238	0.31	0.439	0.629	0.721	0.757	0.45	0.247

TABLE - 2 (cont.)

Mode Rating on (0,1) scale

Respondent No.	FIELDING				SINCERITY				Ability to face adverse situation				BATTING				BOWLING				CAPTAINCY			
	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK	AJ	ST	SG	AK
32	0.873	0.667	0.267	0.414	0.722	0.586	0.563	0.586	0.753	0.619	0.31	0.611	0.563	0.955	0.667	0.171	0.333	0.333	0.551	0.753	0.753	0.619	0.55	0.551
33	0.753	0.619	0.367	0.333	0.619	0.817	0.753	0.551	0.789	0.5	0.618	0.333	0.697	0.899	0.809	0.31	0.31	0.5	0.586	0.563	0.722	0.753	0.59	0.449
34	0.955	0.633	0.333	0.487	0.789	0.619	0.69	0.437	0.667	0.611	0.563	0.282	0.618	0.873	0.753	0.183	0.5	0.3	0.586	0.789	0.764	0.548	0.18	0.5
35	0.773	0.838	0.414	0.5	0.838	0.838	0.838	0.838	0.809	0.367	0.753	0.586	0.773	0.5	0.844	0.31	0.652	0.333	0.721	0.899	0.667	0.838	0.6	0.247
36	0.899	0.667	0.551	0.652	0.873	0.633	0.773	0.633	0.873	0.652	0.697	0.788	0.573	0.955	0.753	0.254	0.333	0.551	0.667	0.955	0.753	0.551	0.5	0.551
37	0.873	0.864	0.5	0.5	0.5	0.854	0.5	0.667	0.786	0.5	0.5	0.746	0.753	0.955	0.817	0.162	0.101	0.452	0.437	0.873	0.449	0.817	0.28	0.183
38	0.873	0.573	0.5	0.697	0.591	0.652	0.773	0.883	0.929	0.633	0.556	0.838	0.69	0.899	0.809	0.5	0.586	0.5	0.753	0.899	0.551	0.838	0.5	0.396
39	0.864	0.633	0.382	0.5	0.721	0.633	0.5	0.721	0.586	0.619	0.382	0.5	0.667	0.753	0.667	0.247	0.449	0.5	0.667	0.789	0.5	0.667	0.28	0.618
40	0.829	0.789	0.561	0.045	0.071	0.955	0.619	0.5	0.556	0.564	0.5	0.5	0.667	0.955	0.667	0.101	0.551	0.563	0.551	0.955	0.551	0.955	0.38	0.247
41	0.809	0.378	0.171	0.117	0.789	0.563	0.753	0.156	0.809	0.31	0.563	0.222	0.652	0.722	0.652	0.188	0.278	0.278	0.449	0.578	0.721	0.449	0.45	0.212
42	0.817	0.563	0 333	0.278	0.722	0.619	0.551	0.348	0.789	0.586	0.382	0.333	0.633	0.789	0.449	0.31	0.303	0.278	0.586	0.586	0.789	0.586	0.33	0.222
43	0.929	0.667	0.5	0.211	0.773	0.955	0.652	0.838	0.789	0.955	0.721	0.789	0.753	0.955	0.789	0.127	0.69	0.817	0.633	0.817	0.753	0.955	0.69	0.838
44	0.773	0.667	0.31	0.31	0.955	0.955	0.817	0.955	0.773	0.722	0.551	0.551	0.619	0.955	0.753	0.127	0.279	0.367	0.5	0.586	0.667	0.789	0.3	0.3
45	0.929	0.563	0.573	0.171	0.586	0.548	0.789	0.437	0.955	0.619	0.746	0.394	0.955	0.955	0.955	0.211	0.563	0.171	0.5	0.9	0.753	0.667	0.62	0.31
46	0.873	0.722	0.162	0.414	0.561	0.561	0.561	0.513	0.809	0.69	0.556	0.449	0.618	0.873	0.652	0.211	0.162	0.5	0.551	0.844	0.5	0.817	0.3	0.5
47	0.873	0.873	0.69	0.69	0.873	0.873	0 667	0.69	0.844	0.844	0.667	0.586	0.778	0.944	0.722	0.348	0.561	0.561	0.809	0.829	0.789	0.899	0.63	0.586
48	0.955	0.667	0.551	0.247	0.367	0.955	0.667	0.873	0.753	0.929	0.596	0.414	0.838	0.955	0.667	0.414	0.191	0.487	0.652	0.955	0.247	0.955	0.57	0.844
49	0.955	0.722	0.5	0.367	0.586	0.955	0.955	0.955	0.69	0.838	0.551	0.551	0.619	0.955	0.817	0.183	0.279	0.5	0.5	0.873	0.214	0.619	0.33	0.333
50	0.899	0.697	0.5	0.437	0.722	0.854	0.563	0.513	0.829	0.667	0.563	0.31	0.551	0.899	0.753	0.343	0.404	0.586	0.69	0.348	0.722	0.817	0.5	0.487
51	0.955	0.513	0.162	0.278	0.955	0.69	0.746	0.69	0.955	0.5	0.817	0.551	0.753	0.955	0.873	0.191	0.31	0.31	0.789	0.899	0.955	0.561	0.82	0.551
52	0.955	0.619	0.487	0.279	0.838	0.817	0.427	0.69	0.817	0.873	0.279	0.586	0.586	0.955	0.873	0.211	0.382	0.414	0.551	0.722	0.817	0.619	0.5	0.303
53	0.955	0.955	0.191	0.382	0.955	0.667	0 789	0.247	0.955	0.563	0.563	0.127	0.5	0.955	0.721	0.056	0.162	0.202	0.414	0.414	0.5	0.618	0.28	0.045
54	0.753	0.563	0.5	0.513	0.721	0.611	0.5	0.667	0.721	0.633	0.5	0.573	0.513	0.757	0.69	0.382	0.382	0.563	0.586	0.753	0.586	0.721	0.44	0.5
55	0.873	0 753	0.618	0.619	0.789	0.899	0.667	0.762	0.722	0.873	0.619	0.619	0.611	0.873	0.667	0.267	0.367	0.333	0.367	0.69	0.697	0.722	0.62	0.5
56	0.899	0.667	0.478	0.5	0.929	0.817	0.619	0.619	0.873	0.789	0.563	0.563	0.753	0.873	0.753	0.367	0.5	0.551	0.667	0.838	0.873	0.789	0.5	0.386
57	0.955	0.753	0.394	0.367	0.439	0.955	0.563	0.829	0.773	0.955	0.5	0.753	0.633	0.955	0.746	0.279	0.247	0.247	0.563	0.955	0.31	0.955	0.62	0.873
58	0.838	0.69	0.367	0.556	0.5	0.955	0.667	0.722	0.838	0.838	0.5	0.69	0.667	0.955	0.753	0.367	0.551	0.333	0.563	0.746	0.5	0.817	0.31	0.5
comb	0.89	0.67	0.33	0.33	0.67	0.9	0.67	0.67	0.8	0.67	0.56	0.57	0.67	0.91	0.78	0.2	0.33	0.4	0.57	0.8	0.67	0.67	0.33	0.33
mean	0.84	0.66	0.39	0.41	0.68	0.78	0.65	0.65	0.76	0.67	0.56	0.52	0.65	0.88	0.73	0.27	0.37	0.42	0.55	0.75	0.62	0.74	0.44	0.45

TABLE - 3

Distribution of Inconsistency: Respondent-wise

(combining all attributes)

Respondent No.	fuzziness 0	0.25	0.5	0.75	1	mean	mean	fuzziness 0	0.25	0.5	0.75	1	Respondent No.
1	9	0	12	3	0	0.34	0.16	12	9	3	0	0	30
2	7	15	2	0	0	0.2	0.3	5	11	6	2	0	31
3	8	14	2	0	0	0.19	0.26	6	11	7	0	0	32
4	10	10	3	1	0	0.2	0.14	12	11	1	0	0	33
5	7	6	10	1	0	0.3	0.19	10	10	4	0	0	34
6	21	2	1	0	0	0.04	0.11	13	11	0	0	0	35
7	12	8	4	0	0	0.17	0.14	12	11	1	0	0	36
8	15	6	3	0	0	0.13	0.24	6	13	5	0	0	37
9	8	9	7	0	0	0.24	0.26	6	11	7	0	0	38
10	9	11	4	0	0	0.2	0.15	11	12	1	0	0	39
11	10	9	4	1	0	0.21	0.21	10	9	4	1	0	40
12	19	5	0	0	0	0.05	0.36	3	9	10	2	0	41
13	7	11	6	0	0	0.24	0.19	9	12	3	0	0	42
14	2	16	6	0	0	0.29	0.2	9	11	4	0	0	43
15	5	8	11	0	0	0.31	0.16	11	11	2	0	0	44
16	13	8	3	0	0	0.15	0.27	6	10	8	0	0	45
17	14	8	2	0	0	0.13	0.22	10	7	7	0	0	46
18	9	9	6	0	0	0.22	0.2	9	11	4	0	0	47
19	13	10	1	0	0	0.13	0.11	14	9	1	0	0	48
20	12	12	0	0	0	0.13	0.11	15	7	2	0	0	49
21	9	12	3	0	0	0.19	0.27	5	12	7	0	0	50
22	3	9	9	3	0	0.38	0.16	11	11	2	0	0	51
23	9	9	6	0	0	0.22	0.11	14	9	1	0	0	52
24	12	9	3	0	0	0.16	0.07	18	5	1	0	0	53
25	8	11	5	0	0	0.22	0.18	10	11	3	0	0	54
26	6	14	4	0	0	0.23	0.22	8	11	5	0	0	55
27	6	13	4	1	0	0.25	0.19	11	9	3	1	0	56
28	9	11	3	1	0	0.21	0.19	9	12	3	0	0	57
29	10	10	4	0	0	0.19	0.23	6	14	4	0	0	58

TABLE - 4

Distribution of Inconsistency: Attribute-wise

(combining all respondents)

		fuzziness							fuzziness						
							mean	mean							
		0	0.25	0.5	0.75	1			0	0.25	0.5	0.75	1		
Fielding	AJ	43	14	1	0	0	0.069	0.203	19	31	8	0	0	AJ	Batting
	ST	16	35	5	2	0	0.22	0.069	45	10	3	0	0	ST	
	SG	20	21	17	0	0	0.237	0.211	20	27	11	0	0	SG	
	AK	19	28	11	0	0	0.216	0.22	21	24	12	1	0	AK	
Sincerity	AJ	23	24	11	0	0	0.198	0.177	28	19	11	0	0	AJ	Bowling
	ST	32	20	6	0	0	0.138	0.259	13	31	13	1	0	ST	
	SG	23	26	8	1	0	0.194	0.185	26	23	7	2	0	SG	
	AK	20	26	11	1	0	0.22	0.159	30	20	7	1	0	AK	
Ability to face adverse situation	AJ	38	13	7	0	0	0.116	0.181	24	26	8	0	0	AJ	Captaincy
	ST	19	24	11	4	0	0.25	0.168	29	19	10	0	0	ST	
	SG	17	30	10	1	0	0.228	0.272	13	27	18	0	0	SG	
	AK	10	29	17	2	0	0.297	0.254	15	28	14	1	0	AK	

C.2 Results from the small survey data

Second Survey Data Analysis

Respondent × Attribute Inconsistency Matrix

Respondent No.	Question No 1	2	3
1	1/3	1/3	1/3
2	0	1/3	1/3
3	0	0	0
4	1/3	2/3	1/3
5	0	0	1/3
6	0	0	0
7	0	0	0
8	0	0	1/3
9	0	0	0
10	0	0	1/3

Repeated scale options				I = 0	I =1/3	I = 2/3	I = 1	
3	5	7	9	0.0243	0.2899	0.6709	0.0148	Probability
				0.0243	*0.3143*	*0.9852*	1.0000	Cum. Prob.

Binomial Distribution with n=10	c-values	Probability[X ≥c]		
	0	1	1	1
	1	0.2184	0.9770	1.0000
	2	**0.0234**	0.8717	1.0000
	3	0.0015	0.6544	1.0000
	4	0.0001	0.3888	1.0000
	5	0.0000	0.1758	1.0000
	6	0.0000	0.0587	1.0000
	7	0.0000	**0.0139**	1.0000
	8	0.0000	0.0022	0.9996
	9	0.0000	0.0002	0.9909
	10	0.0000	0.0000	0.8613

H_0= random response

X = no. of respondents with I-values ≤ 0 1/3

Reject H_0 (and conclude that the Qi is ok) if $X \geq c$

Conclusion: only Q3 would be discarded if 0-inconsistency is used as the criterion.
All other Q's in all other situations would be kept.

Inference regarding the Respondents:

Binomial distribution with n=3

c-values	Probability[X ≥ c]		
0	1	1	1
1	***0.0713**	0.6776	1.0000
2	0.0017	0.2342	0.9993
3	0.0000	**0.0310**	0.9562

This tells us that only respondents having all 3 I-measures 0 or 1/3 should be kept
Thus at 3.5% level only respondent 4 is to be discarded.

Going by 0 I-measures:
Respondents with X = 0 would be rejected where X = no. of attributes with I values = 0,
i.e. if he/she is not perfectly consistent for at least one of the 3 attributes, he/she is discarded
That rules out Respondents 1 and 4.

NB: No. of I values ≤ 2/3 (or 1) is not a reasonable TS for screening respondents or attributes

Now turn the table, i.e. switch the null and alternative hypothesis
with p* as in MI set up as 0.75 0.8 0.9 0.95

c-values	Probability[X ≤ c]			
0	0.0000	0.0000	0.0000	0.0000
1	0.0000	0.0000	0.0000	0.0000
2	0.0004	0.0001	0.0000	0.0000
3	0.0035	0.0009	0.0000	0.0000
4	**0.0197**	0.0064	0.0001	0.0000
5	0.0781	**0.0328**	0.0016	0.0001
6	0.2241	0.1209	**0.0128**	0.0010
7	0.4744	0.3222	0.0702	**0.0115**
8	0.7560	0.6242	0.2639	0.0861
9	0.9437	0.8926	0.6513	0.4013
10	1.0000	1.0000	1.0000	1.0000

So only Q3 would be rejected at all these levels of theta, except for theta =0.95 where
even Q2 would be rejected

Binomial distribution with n=3

c-values	Probability[X ≤ c]			
0	**0.0156**	**0.0080**	0.0010	0.0001
1	0.1563	0.1040	**0.0280**	**0.0073**
2	0.5781	0.4880	0.2710	0.1426
3	1.0000	1.0000	1.0000	1.0000

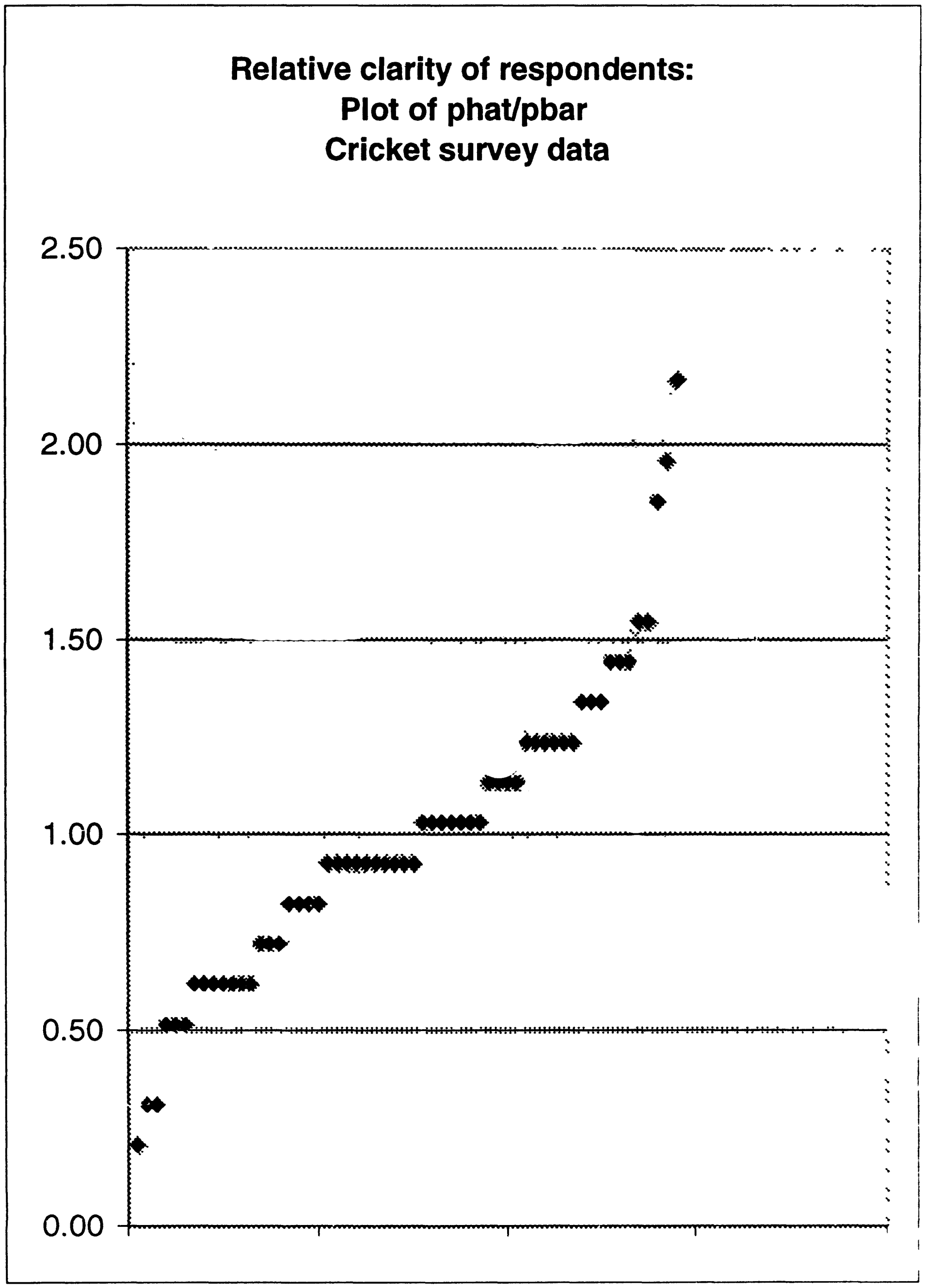
(Again using 0-inconsistencies), Respondent 1 & 4, with no perfect consistency would be discarded at $\theta = 0.75$ and 0.8. And at $\theta = 0.9$ and 0.95, Respondent 2 will be discarded, in addition.

---

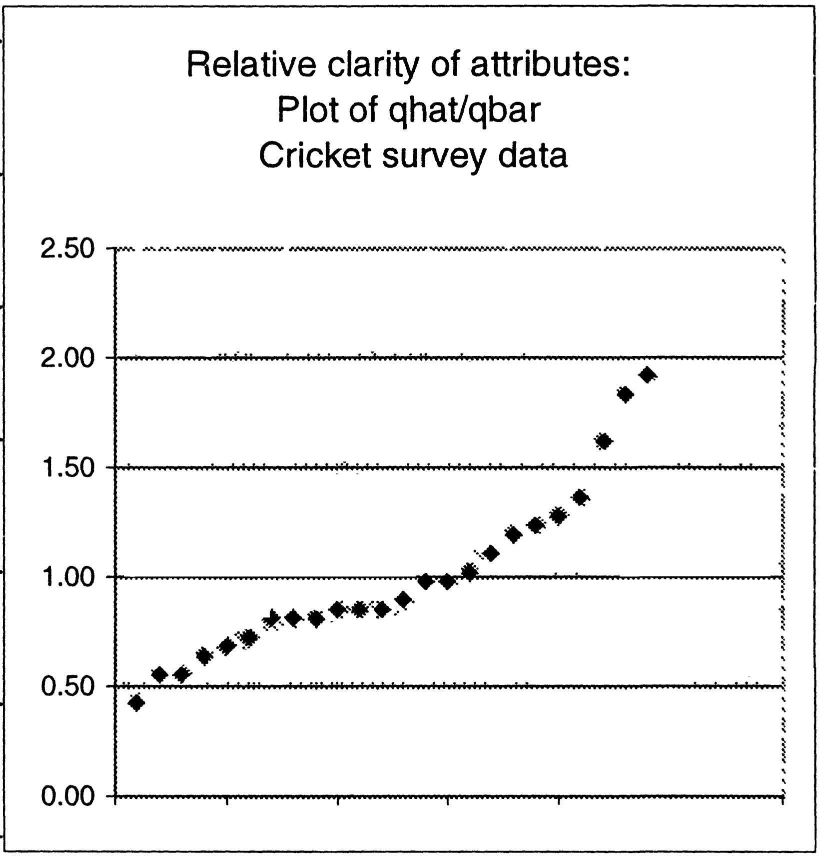
* NB: In all decisions, except here, the significance level is chosen at closest to, but not exceeding 0.05.

D Data analysis and Plots using Product Model Approach

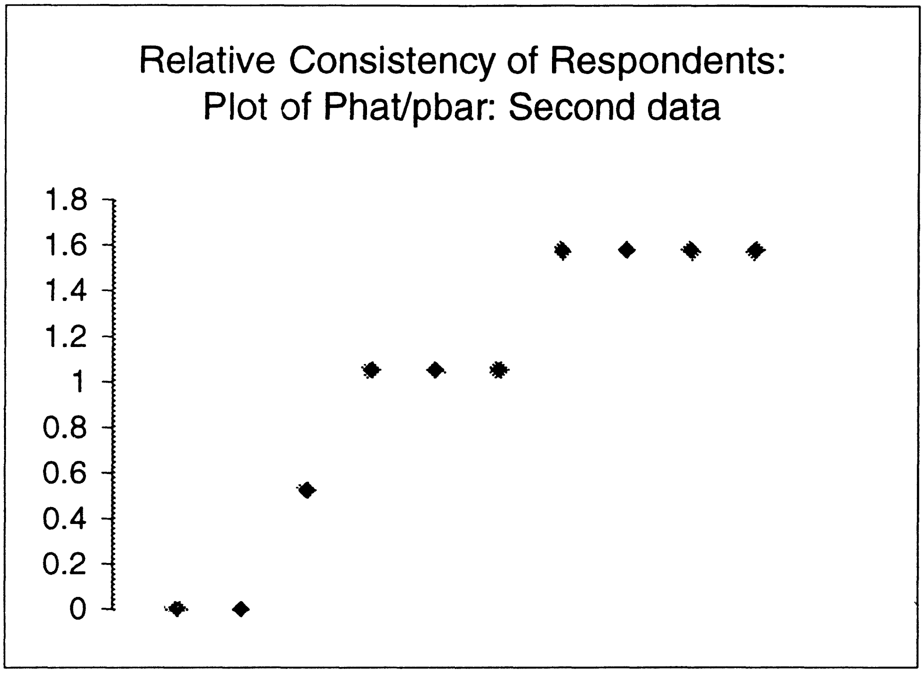
Res. No.	p_i_hat	phat/pbar
14	0 08	***0.21***
22	0 13	***0.31***
41	0 13	***0.31***
15	0 21	0 52
31	0 21	0 52
50	0 21	0 52
26	0 25	0 62
27	0 25	0 62
32	0 25	0 62
37	0 25	0 62
38	0 25	0 62
45	0 25	0 62
58	0 25	0 62
2	0 29	0 72
5	0 29	0 72
13	0 29	0 72
3	0 33	0 82
9	0 33	0 82
25	0 33	0 82
55	0 33	0 82
1	0 38	0 93
10	0 38	0 93
18	0 38	0 93
21	0 38	0 93
23	0 38	0 93
28	0 38	0 93
42	0 38	0 93
43	0 38	0 93
47	0 38	0 93
57	0 38	0 93
4	0 42	1 03
11	0 42	1 03
29	0 42	1 03
34	0 42	1 03
40	0 42	1 03
46	0 42	1 03
54	0 42	1 03
39	0 46	1 13
44	0 46	1 13
51	0 46	1 13
56	0 46	1 13
7	0 50	1 24
20	0 50	1 24
24	0 50	1 24
30	0 50	1 24
33	0 50	1 24
36	0 50	1 24
16	0 54	1 34
19	0 54	1 34
35	0 54	1 34
17	0 58	1 44
48	0 58	1 44
52	0 58	1 44
8	0 63	1 55
49	0 63	1 55
53	0 75	1 85
12	0 79	1 96
6	0 88	2 16
pbar	0 40	



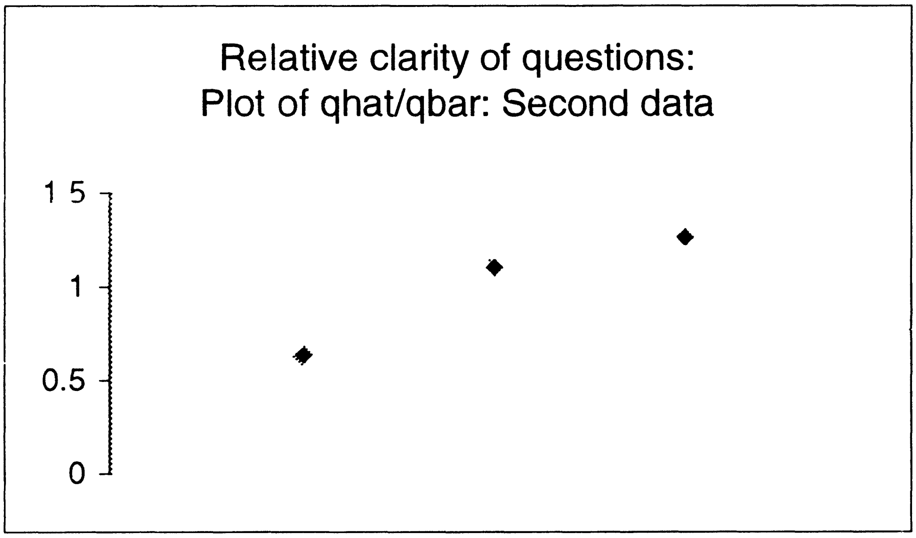
	attribute	q_j_hat	qhat/qbar
FIELDING	AJ	0.74	1.83
	ST	0.28	0 68
	SG	0.34	0 85
	AK	0.33	0.81
SINCERITY	AJ	0.40	0.98
	ST	0.55	1.36
	SG	0.40	0.98
	AK	0.34	0.85
in adv sit	AJ	0.66	1.62
	ST	0.33	0.81
	SG	0.29	0.72
	AK	0.17	***0.43***
BATTING	AJ	0.33	0 81
	ST	0.78	1.92
	SG	0.34	0.85
	AK	0.36	0.90
BOWLING	AJ	0.48	1.19
	ST	0.22	0.55
	SG	0.45	1.11
	AK	0.52	1.28
CAPTAINCY	AJ	0.41	1 02
	ST	0.50	1.24
	SG	0.22	0.55
	AK	0.26	0 64
	q-bar	0.40	



phat/pbar	Respondent No
0	1
0	4
0 526316	2
1 052632	5
1 052632	8
1 052632	10
1 578947	3
1 578947	6
1 578947	7
1 578947	9



qhat/qbar	Question No
0 631579	3
1 105263	2
1 263158	1



E Cumulative Binomial Tables Needed in Section 5.1

	n=	24	
	0.0032	0.069	0.5013
x	PROB(X >= x)		
0	1	1	1
1	0.0740	0.8202	1.0000
2	**0.0027**	0.5004	1.0000
3	0.0001	0.2278	1.0000
4	0.0000	0.0796	0.9999
5	0.0000	0.0220	0.9993
6	0.0000	**0.0049**	0.9968
7	0.0000	0.0009	0.9890
8	0.0000	0.0001	0.9689
9	0.0000	0.0000	0.9260
10	0.0000	0.0000	0.8493
11	0.0000	0.0000	0.7336
12	0.0000	0.0000	0.5856
16	0.0000	0.0000	0.0776
17	0.0000	0.0000	0.0329
18	0.0000	0.0000	0.0117
19	0.0000	0.0000	**0.0034**
20	0.0000	0.0000	0.0008
21	0.0000	0.0000	0.0001
22	0.0000	0.0000	0.0000
23	0.0000	0.0000	0.0000
24	0.0000	0.0000	0.0000

n=		24		
theta	0.75	0.8	0.9	0.95
x	PROB(X <= x)			
6	0.0000	0.0000	0.0000	0.0000
7	0.0000	0.0000	0.0000	0.0000
8	0.0000	0.0000	0.0000	0.0000
9	0.0001	0.0000	0.0000	0.0000
10	0.0005	0.0000	0.0000	0.0000
11	0.0021	0.0002	0.0000	0.0000
12	0.0072	0.0010	0.0000	0.0000
13	**0.0213**	0.0038	0.0000	0.0000
14	0.0547	0.0126	0.0001	0.0000
15	0.1213	**0.0362**	0.0003	0.0000
16	0.2338	0.0892	0.0017	0.0000
17	0.3926	0.1889	0.0075	0.0001
18	0.5778	0.3441	**0.0277**	0.0010
19	0.7534	0.5401	0.0851	0.0060
20	0.8850	0.7361	0.2143	**0.0298**
21	0.9602	0.8855	0.4357	0.1159
22	0.9910	0.9669	0.7075	0.3392
23	0.9990	0.9953	0.9202	0.7080
24	1.0000	1.0000	1.0000	1.0000

	n=	58	
	0.0032	0.069	0.5013
x	PROB(X >= x)		
0	1	1	1
1	0.1696	0.9842	1.0000
2	**0.0150**	0.9162	1.0000
3	0.0009	0.7726	1.0000
4	0.0000	0.5739	1.0000
5	0.0000	0.3715	1.0000
6	0.0000	0.2094	1.0000
7	0.0000	0.1033	1.0000
8	0.0000	0.0449	1.0000
9	0.0000	0.0173	1.0000
10	0.0000	**0.0060**	1.0000
11	0.0000	0.0019	1.0000
12	0.0000	0.0005	1.0000
35			0.0768
36			0.0453
37			0.0251
38			0.0130
39			0.0063
40			0.0028
41			**0.0012**
42			0.0005
43			0.0002

	n=	58	
theta	0.8	0.9	0.95
x	PROB(X <= x)		
39	0.0153	0.0000	0.0000
40	**0.0310**	0.0000	0.0000
41	0.0585	0.0000	0.0000
42	0.1031	0.0001	0.0000
43	0.1695	0.0005	0.0000
44	0.2599	0.0014	0.0000
45	0.3725	0.0042	0.0000
46	0.4998	0.0112	0.0000
47	0.6298	**0.0273**	0.0001
48	0.7490	0.0605	0.0006
49	0.8462	0.1215	0.0022
50	0.9163	0.2203	0.0080
51	0.9602	0.3597	**0.0252**
52	0.9839	0.5287	0.0691
53	0.9946	0.7008	0.1636
54	0.9986	0.8443	0.3297
55	0.9997	0.9382	0.5594
56	1.0000	0.9835	0 7931
57	1.0000	0.9978	0.9490
58	1.0000	1.0000	1.0000