Stability Issues in Supply Chain Networks: Implications for Coordination Mechanisms

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With supply chains evolving into complex networks of diverse and independent economic entities, supply chain coordination has emerged as a key management capability for improving overall performance of the network. For long term sustainability, it is important that such networks remain stable with little incentive for players to sever their linkages, especially in supply chains with participative constraints on their members. While topic of coordination has received vast attention from both academia and practitioners, the issues related to network stability have not been addressed adequately. In order to bridge this gap in the literature, we focus on a two tier supply chain network competing with independent producers. The network involves a marketing agent that coordinates activities among the network producers via a price and profit sharing based coordination mechanism. We develop a (noncooperative) game theoretic model and analyze the impact of the coordination mechanism on stability of the network. In particular, we focus on three factors: cost parameters, number of producers vis-à-vis level of competition, and profit sharing. We derive response functions for the players involved and show that optimal decisions by coordination leads to Nash equilibrium for the supply chain. In addition, we develop structural results to characterize stability of the network. Our results show that while profit sharing parameter of the coordination mechanism has no impact on network surplus, it has implications for network stability. These results bring out relationship between the factors of interest and provide insights for determining the decision parameters of the coordination mechanism.

Key words: network stability; coordination; competition; game theory

1. Introduction

During the past two decades, supply chains have evolved into complex networks involving diverse players, each with its own rights and individual interests. In this environment, supply chain coordination with a focus on mechanisms to align decisions of members has emerged as a key management capability for improving the overall effectiveness of the network (see Cachon 2003, Chen 2003, Li and Wang 2007). For long term sustainability, it is important that such networks remain stable with little incentive for players to sever their linkages. The issue is particularly important in networks with exclusivity contracts that restrict players from participating simultaneously in supply chains of similar or competing products. Franchises, exclusive dealerships, captive facilities (such as manufacturing plants, distribution units, etc.) and cooperatives are typical examples of supply chain networks that exhibit some of these features. The presence of competing supply chains often provides players in the network with alternate avenues that may undermine the stability of the network. While coordination issues have received much attention in supply chain management literature (see Whang 1995, Tsay et al. 1998, Cattani et al. 2003, Kouvelis et al. 2006), issues related to stability have not been studied in detail. In this paper we aim to bridge this gap and develop a modeling approach to study network stability issues and examine the impact of coordination mechanisms on stability of supply chain networks.

The motivation for our work comes from mixed results in the cooperative sector of India. Over the last six decades, *AMUL*, a milk producers' cooperative, has a led dairy revolution in India that has resulted in India becoming one of the largest producers of milk in the world (Bellur et al. 1990). The success of AMUL is achieved within the framework of a network of cooperatives organized in a hierarchical manner. The network structure has been subsequently replicated in various other sectors - such as oil, sugar, wheat, fertilizer, etc. However, similar success has eluded many of these cooperatives, and in some cases the networks have disintegrated (Bandyopadhyay 1996, Das et al. 2006). Recently, the AMUL network has come under strain with competition from private players, and consequently, alternate avenues - such as changing supply chain alliances, severing linkages with the existing network, establishing independent production units, etc. - have evolved for the players in the network (Chandra and Tirupati 2003). Similar competitive setting can be observed in network structures such as Independent Grocers Association (IGA) - a group of independent retailers (www.iga.com), Unified Western Grocers Inc. - a purchasing cooperative of independent grocers (www.unifiedgrocers.com), and European Social Franchising Network, CAP Market in Germany - cooperatives of sheltered workshops (www.socialfranchising.coop, www.cap-markt.de).

Specifically, in this paper we examine stability issues in a two-tier supply chain network comprising of several producers operating in a competitive market. While some of the producers operate independently and offer their product in the market directly, the rest form a cooperative network (hereafter referred to as network) and supply through a marketing agent that acts as a coordinator. Production decisions by the network producers are influenced by the coordination mechanism used for sharing the revenues generated. In this paper we analyze a profit sharing based mechanism that is popular both in practice (see Azfar and Danninger 2001, Heywood and Jirjahn 2009) and the literature (see Chen et al. 2001, Foros et al. 2009). The coordination mechanism involves procurement price paid by marketing agent to the network producers and surplus sharing. We develop a game theoretic model to describe the problem context and characterize the decisions of both network and independent producers. Our development involves integration of (i) principles of coordination from supply chain management literature, and (ii) literature in economics and industrial organization on network stability. We derive response functions for the players involved and show that optimal decisions by coordination leads to Nash equilibrium for the supply chain. For instance, we show that there exists a range of procurement price in which both network and independent producers compete together. For procurement price below a threshold value, the network producers do not compete. Similarly, for procurement price beyond a threshold, the independent producers do not compete. In addition, we develop structural results to characterize stability of the network. Our results show that while profit sharing parameter of the coordination mechanism has no impact on network surplus, it has implications for network stability. These results bring out relationship between the factors of interest and provide insights for determining the decision parameters of the coordination mechanism.

The remainder of the paper is organized as follows. In Section 2, we review the relevant literature and position the work described in this paper. The problem context and model development are described in Section 3. Section 4 covers model analysis. Section 5 discusses network stability. We mention the key findings and conclude the paper in Section 6.

2. Literature Review

As mentioned earlier, the literature related to the problem considered in this paper comes from two streams: (i) operations and supply chain management, in particular on coordination, and (ii) network formation and stability in economics and industrial organization. In what follows we present a brief overview of the literature and relate it to the issues addressed by our work in the paper.

2.1. Supply Chain Coordination

In the past two decades, coordination has emerged as a new paradigm for managing and improving significantly performance of supply chains and has attracted growing attention from both academia and practitioners. The literature on the subject is quite extensive with multiple perspectives to examine a wide range of issues employing diverse methodologies. As a result, depending on the context, the term coordination has been interpreted in different ways and several alternative definitions have been offered. For example, Malone and Crowston (1990, 1994) define coordination as "*the act of managing interdependencies between activities performed to achieve a common goal*". Similarly, Lambert et al. (1998) define coordination as a process of planning, executing and controlling the interdependencies of activities carried out by different supply chain members or business units in order to create value for the end customer. According to Desai et al. (2004), supply chain coordination involves mitigating channel inefficiencies of decentralized system and achieving the same profit as a centralized system.

The literature on supply chain coordination may be broadly classified into the following categories based on methodology/approach:

(i) Empirical studies that focus on assessing extent of coordination in supply chain and its impact on performance.

Such studies typically identify the adverse impact of lack of coordination on performance and benefits of coordination and focus on identifying factors that facilitate coordination. Examples of such studies can be found in Stank et al. (1999), Stock et al. (2000), Kulp et al. (2004), Peterson et al. (2005). In the food and consumer packaged goods industry, Kulp et al. (2004) observe that various integration techniques are directly and positively associated with performance measures. Peterson et al. (2005) show that coordinating supplier capabilities in product design process results in improved financial performance.

(ii) Studies using approaches for social systems/organizational settings with focus on inter-organization and inter-functional issues based on trust and social mechanisms. For instance, Jones et al. (1997) observe that macro-culture (common values, norms, beliefs, etc.) facilitates efficient exchange among the parties and reduces coordination costs by increasing the ease of exchanging customized goods or services among autonomous parties. Handfield and Bechtel (2002) note that buyer-dependence, supplier human asset investments, and trust are all positively associated with improved supply chain responsiveness.

(iii) Modeling approaches with focus on decision making in supply chains.

The literature in this area deals with issues related to coordinated planning for optimizing integrated supply chain (see Thomas and Griffin 1996, Swaminathan and Tayur 2003, Li and Wang 2007), coordination contracts for decentralized systems (see Tsay et al. 1998, Cachon 2003), information sharing among the players (see Chen 2003, Huang et al. 2003), etc.

The literature on supply chain coordination is clearly quite vast and a comprehensive review is beyond the scope of this paper. Instead, we provide a brief overview of literature related to design and analysis of coordination mechanisms in supply chains that is perhaps most relevant to the work presented in this paper. The focus of research in this body of literature is on decentralized supply chains with decision rights for individual entities. Typically, two players are considered at a time (manufacturer-retailer, supplier-manufacturer, etc.) and contractual terms define the coordination mechanism. Integrated supply chain with optimal decision for the entire system is considered as the benchmark and the objective is to analyze and characterize performance of the entire system with reference to this benchmark. In this context, according to Cachon and Kok (2010), a supply chain is coordinated if the combined profit of the supply chain members is maximized under decentralized decision making. Similarly, a contract is said to coordinate the supply chain if the set of supply chain optimal actions is a Nash equilibrium, i.e., no firm has a profitable unilateral deviation from the set of supply chain optimal actions (Cachon 2003).

The coordinating contracts documented in the supply chain literature are numerous. Review of this literature can be found in Tsay et al. (1998) and Cachon (2003). The contracts that are primarily studied under a variety of supply chain environment are: revenue-sharing (Mortimer 2002, Cachon and Lariviere 2005), whole-sale price (Bernstein et al. 2006), mix of both revenue-sharing and whole-sale price (Pasternack 2002), quantity flexibility (Tsay 1999), buy-back (Pasternack 1985), sales-rebate (Taylor 2002), price-discount (Bernstein and Federgruen 2005), two-part tariff (Raju and Zhang 2005), contracts with options (Schuster et al. 2002), etc. These studies discuss flexibility, applicability and efficiency of the contracts in supply chains. Cachon and Lariviere (2005) show how some of these contracts are interrelated and sometimes how they are identical. However, bulk of this research deals with linear prices under a variety of contractual conditions and/or environment (see Cho and Gerchak 2001, Chen and Federgruen 2001). The primary results relate to conditions under which coordination is achieved and deviation from optimal performance of the supply chain. For instance, in a standard newsvendor problem, Lariviere and Porteus (2001) show that the price only contract does not coordinate the supply chain as the retailer does not order enough inventory to maximize the supply chain's profit. However, coordination is possible under buy back contract (Pasternack 1985) and quantity discount contract (Tomlin 2003).

Limited research has been reported on coordination mechanisms that involve profit/risk sharing along with linear prices. For example, Lariviere and Porteus (2001) model the risk sharing behavior in a newsboy problem using price-only contracts and obtain the conditions in which supply chains are coordinated. Jeuland and Shugan (1983) demonstrate how profit sharing can coordinate a supply chain. Foros et al. (2009) examine the profit sharing based coordination mechanism in the context of the Norwegian telecommunications industry. In their paper, Foros et al. (2009) show how the mechanism helps the upstream firm in preventing destructive competition between downstream firms that produce relatively close substitutes. Chen et al. (2001) show how the profit sharing based mechanism coordinates a distribution channel when demand is uncertain. Heywood and Jirjahn (2009) show the interdependence of firm size and profit sharing for different types of technologies.

Azfar and Danninger (2001) show how employment stability and wage growth can be influenced via profit sharing.

To summarize, the focus of literature on coordination mechanism in supply chain is on efficiency with the objective of developing mechanisms that achieve optimal performance for the entire supply chain. The sustainability and integrity of the supply chain is implicit and thus stability issues have not received much attention. (It may be noted that stability issues considered in Lee et al. (1997), Rinddalls and Bennett (2002), Daganzo (2004), Ouyang (2007) and many others are in the context of bullwhip effect that relates to supply chain performance and different from sustainability issues that we consider in this paper.) While supply chain stability is not a major factor for supply chains with no constraints on participation, it is an important issue for a class of supply chains with participative restrictions. As described earlier, cooperatives, exclusive dealerships, franchises, etc. are examples of supply chains with exclusivity contracts for the members that forbid member participation in competing supply chains. We address this gap in the literature and develop a game theoretic model to examine related issues and devise structural results to characterize stability of such systems and provide managerial insights. In the following section, we review related literature from economics.

2.2. Network Stability

As mentioned earlier, there exists extensive literature in economics devoted to the study of networks. While much of this research focuses on network formation and application of game theoretic approaches to study existence and properties of equilibria, some recent work in the area addresses stability issues in a variety of networks. It may also be noted that the concept of stability is not unique and several variants of stability have been rendered in the literature. For example, Jackson and Wolinsky (1996) introduce the notion of pairwise stability based on individual incentives for the players in maintaining or deviating from the existing linking structure in the network. Similarly, Dutta and Mutuswami (1997) and Jackson and van den Nouweland (2005) define and study strongly and weakly stable networks based on Nash equilibria. A comprehensive review of this literature is rather extensive and beyond the scope of this paper. Instead we present a brief overview of literature that is relevant in the context of the problem considered in this paper. Detailed discussion on the literature on network formation and stability can be found in Jackson (2003, 2005), Bloch and Jackson (2006) and the references therein.

A network is pairwise stable if no player would benefit from severing an existing link, and no two players will benefit by forming a new link. Accordingly, this stability concept considers deviations on a single link at a time. In our problem, the producers in the market are independent economic entities, and their decision on network participation is purely based on own interests. By the individual decision making aspect of the producers in the market, the notion of pairwise stability can be adopted to address the stability issues in our model. Pairwise stability is too weak condition for modeling network formation, however, it is a necessary condition for stronger stability notions (Calvó-Armengol 2003). Due to its computational (relative) simplicity and its ability to generate sharp predictions in many contexts, pairwise stability concept has been widely adopted in applied studies. Many social and economic network models, such as connections model, co-author model (Jackson and Wolinsky 1996), spatial connections model (Johnson and Gilles 2000), buyer-seller networks (Kranton and Minehart 2000, 2001), free-trade networks (Furusawa and Konishi 2007), communication networks (Slikker and van den Nouweland 2000), resource sharing and cost reduction (Goyal and Joshi 2003), etc. are analyzed using pairwise stability.

The approach used in Shaffer (1995), Bloch (2002) and Escrihuela-Villar (2002) in the context of sustainability and stability of cartels is similar to the one used in our paper. Shaffer (1995) studies stability issues in a competitive market with two types of players; a cartel with power to enforce its decision on its members and fringe players (not members of the cartel). The problem is modeled using the frameworks of Cournot and Bertrand type simultaneous-moves and Stackelberg kind leader-follower. The cartel is assumed to behave as a monopolist. The main results of the paper relate to characterization of relative sizes of the cartel and fringe players for stability of the network. Escrihuela-Villar (2002) studies how the participation incentive interacts with the incentive to maintain a collusive agreement. The main result is that collusion is easier to sustain in the larger coalition. Firms will be more willing to participate because otherwise, they risk that collusion completely collapses, as remaining members are unable to sustain collusion. We find these results apply under special conditions in our study. Bloch (2002) models cooperation based on collusive agreements and cost-reducing alliances. The main results are: (i) all firms prefer to stay outside the coalition (cartel) and free-ride on the public good provided by the coalition (cartel) members in a simultaneous link formation framework, and (ii) in a bilateral collusive network, there are exactly two pairwise stable networks: the empty network and the complete network. The author also finds that profits of each firm in a stable coalition are less than those of an associated Cournot fringe firms. In our context, we show that the result is not necessarily valid.

While there are similarities between our approach and that of Shaffer (1995), Bloch (2002) and Escrihuela-Villar (2002), our focus and the problem context are different. For example, unlike the earlier work, we explicitly model and consider the cost differences between the two types of players and study their impact on stability of the network. Similarly, we do not assume an enforcement mechanism as in Shaffer (1995) and Bloch (2002). Instead we explicitly include in the model coordination mechanism to reflect operational reality of the network. More generally, the economics literature ignores operational details and the analysis and results are based on exogenously specified allocations rules and value functions. In our context, we focus on the operational details and endogenous value functions. Hence, in addition to the impact of the relative size of the two groups, we focus on the cost differences, and the parameter choices of the coordination mechanism and determine their impact on stability. In the process, we characterize the equilibrium production quantity and profits of the different players as well.

3. Model Building

Motivated by the AMUL environment described in Section 1, in this paper we examine stability issues in a two-tier supply chain for a single product with n producers. In order to focus on the key factors affecting stability, we assume that the producers are identical with unit production cost and ignore capacity constraints. We consider the supply chain in which m of the producers (referred to as independent producers) supply the market directly. The remaining $(n-m)$ producers (referred to as network producers) form a network and supply the market through a marketing agent (Refer Figure 1). We assume that the independent producers and the network producers incur marketing and distribution costs of v_{IP} and v_{NP} per unit, respectively. This is in addition to the constant manufacturing cost of c per unit. We assume that $v_{IP} > v_{NP}$ to reflect the benefits realized through economies of scale and cooperative behavior of the network producers. We also assume that there are no fixed costs associated with production, sales and distribution functions of the players.

We consider a competitive market with undifferentiated product and downward sloping demand curve. We assume that the market price is given by $p\,(q) = a - bq$, where q is the total supply in the market, a represents the limiting price (beyond which there is no demand), and b, the rate of price change with respect to the market supply, denotes the price sensitivity to the supply quantity.

Besides performing the distribution and selling functions for the network producers the marketing agent acts as a coordinator for aligning the producers' decisions with the network objectives. In this paper we consider a coordination mechanism, a price and profit sharing based coordination mechanism, that is similar to the one practiced in our motivating example and popular in the literature as well. The mechanism involves two parameters: (i) procurement price w offered by the marketing agent to the network producers, and (ii) α, the fraction of the marketing agent's surplus (surplus net of cost of sales and distribution, and procurement) distributed among the network producers. The surplus is distributed among the network producers in proportion of the quantity

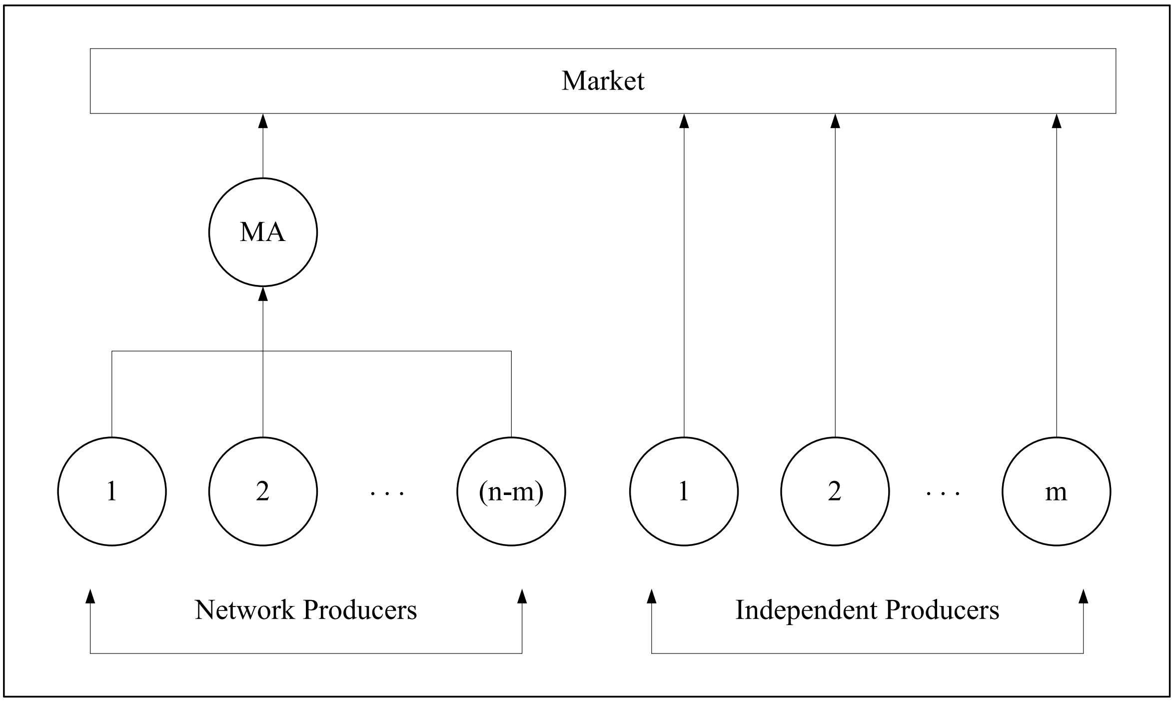

Figure 1 Network Structure

supplied. The remainder of the surplus is retained by the marketing agent. It may be noted that this coordination mechanism is similar to the ones studied in Azfar and Danninger (2001), Chen et al. (2001), Foros et al. (2009) and Heywood and Jirjahn (2009) and described earlier in Section 2. In our model, we assume that the network's objective is to maximize the surplus generated for the network producers, and hence, the marketing agent's role is to choose the parameters (w, α) consistent with this objective. The marketing agent announces these parameters and is required to accept the entire supply from the network producers. We further assume that all producers - both network and independent - act in their respective interests and respond accordingly to the marketing agent's choice of the parameters of the coordination mechanism.

We assume that the cost parameters, the parameters of the coordination mechanism and the demand function are common knowledge. We now present the decision problem for these three kinds of players: network producers, independent producers, and the marketing agent. The complete notation used is summarized in Table 1.

Let π_i, π_j, and π_N be the net surplus for network producer i, independent producer j, and the entire network, respectively.

The network producer i's problem, NP_i, is given by:

Parameter	Description
n	Number of producers in the market
m	Number of independent producers ($m < n$)
$(n-m)$	Number of network producers
i (or k)	Network producer i (or k); $i, k = 1, 2, ..., (n-m)$
j (or l)	Independent producer j (or l); $j, l = 1, 2, ..., m$
q_i	Quantity produced by network producer i
q_j	Quantity produced by independent producer j
ϕ_i	Portion of network producer i in the profit shared by the marketing agent; $\phi_i = q_i / \sum_k q_k$
q	Total supply of the product in the market; $q = \sum_{i=1}^{n-m} q_i + \sum_{j=1}^{m} q_j$
p	Marketing clearing price of the product; $p(q) = a - bq$
q_{-i}	$\sum_{k \neq i} q_k + \sum_j q_j$
q_{-j}	$\sum_i q_i + \sum_{l \neq j} q_l$
w	Per unit procurement price for the product offered by the marketing agent to the network producers
α	Fraction of the surplus shared by the marketing agent with the network producers; $\alpha \in [0,1]$
c	Producer's marginal cost of production
v_{NP}	Marketing agent's marginal cost of sales and distribution of the product
v_{IP}	Independent producer's marginal cost of sales and distribution of the product
π_i	Profit function of network producer i
π_j	Profit function of independent producer j
π_N	Profit function of the supply chain network
π_{MA}	Profit function of the marketing agent

Table 1 **Notation**

$$\begin{aligned} NP_i : \pi_i (w, \alpha, q_{-i}) &= \max_{q_i \geq 0} (w - c) q_i + \alpha \phi_i \left\{ [p(q) - w - v_{NP}] \sum_{k=1}^{n-m} q_k \right\} \\ &= \max_{q_i \geq 0} [(1-\alpha) w + \alpha (a - v_{NP}) - c - \alpha b (q_i + q_{-i})] q_i \end{aligned} \tag{1}$$

The independent producer j's, IP_j, is given by:

$$\begin{aligned} IP_j : \pi_j (w, \alpha, q_{-j}) &= \max_{q_j \geq 0} [p(q) - c - v_{IP}] q_j \\ &= \max_{q_j \geq 0} [a - c - v_{IP} - b(q_j + q_{-j})] q_j \end{aligned} \tag{2}$$

The marketing agent's problem, MA, is given by:

$$MA: \pi_N(q) = \max_{\substack{w\geq 0\\ 0\leq\alpha\leq 1}} \left[p(q) - c - v_{NP}\right] \sum_{k=1}^{n-m} q_k$$
$$= \max_{\substack{w\geq 0\\ 0\leq\alpha\leq 1}} \left[a - c - v_{NP} - bq\right] \sum_{k=1}^{n-m} q_k \tag{3}$$

An alternative objective for the marketing agent could be to maximize the agent's profit. Hence, the marketing agent's problem can be formulated as follows:

$$\pi_{MA}(q) = \max_{\substack{w\geq 0\\ 0\leq\alpha\leq 1}} (1-\alpha)\left[p(q) - c - v_{NP}\right] \sum_{k=1}^{n-m} q_k$$
$$= \max_{\substack{w\geq 0\\ 0\leq\alpha\leq 1}} (1-\alpha)\left[a - w - v_{NP} - bq\right] \sum_{k=1}^{n-m} q_k$$

However, as mentioned earlier, we consider the marketing agent's problem as given in (3).

In the remainder of this paper, we make the following assumptions about the relative costs of the network and independent producers.

Assumption 1. $a - c - v_{IP} \geq 0$

Assumption 2. $a - c - v_{NP} \geq 0$

Assumption 3. $v_{IP} > v_{NP}$

The assumptions above are motivated by the problem context. It may be noted that if the assumptions do not hold, the problem is not meaningful and/or may be solved trivially. For example, Assumptions 1 and 2 are necessary for both players to compete. As mentioned earlier, Assumption 3, is a consequence of economies realized by cooperation among the network producers and there is incentive to form a network if the assumption does not hold.

We conclude by noting that the objective functions of the problems NP_i and IP_j are concave in their respective arguments. The objective function of the problem MA is also concave in its argument which is shown in next section. Further, the problem structure is a Stackelberg kind of sequential game with marketing agent as the leader. With parameters w and α given (announced by the marketing agent), the producers' decisions in the second stage follow a Cournot type game.

Below we present the analysis of the model for $\alpha \in (0,1)$. The two special cases with $\alpha = 0$ and $\alpha = 1$ are discussed in Section 5. For brevity, we drop the arguments - such as α - whenever they are obvious.

4. Model Analysis

The network producers and the independent producers respond to the marketing agent's offer in the price and profit sharing based coordination mechanism. We take the backward induction based solution approach to obtain the subgame equilibrium of the game between the marketing agent and the producers in the market. First, we characterize the producers' decisions as a subgame-perfect Nash equilibrium of the of the simultaneous-move type Cournot game in which each producer maximizes its profit as a response to the total production of the other producers in the market and the marketing agent's offer. Further, we analyze the marketing agent's problem by incorporating the Nash equilibrium of the producers' response functions and characterize the equilibrium of the game.

4.1. Producer's Problem

First, we characterize in Lemma 1 below the response function for each type of producer. The response function describes the optimum production for the producer, given the quantity supplied by the other producers in the system.

LEMMA 1. *The response functions for the network producers and the independent producers are as follows:*

$$q_i^*(w, q_{-i}) = \begin{cases} \dfrac{f_i(w, q_{-i})}{2\alpha b} & \textit{if } f_i(w, q_{-i}) > 0 \\ 0 & \textit{otherwise} \end{cases} \tag{4}$$

$$q_j^*(w, q_{-j}) = \begin{cases} \dfrac{f_j(w, q_{-j})}{2b} & \textit{if } f_j(w, q_{-j}) > 0 \\ 0 & \textit{otherwise} \end{cases} \tag{5}$$

where,

$$f_i(w, q_{-i}) = (1-\alpha)\, w + \alpha\,(a - v_{NP}) - c - \alpha b q_{-i} \tag{6}$$

$$f_j(w, q_{-j}) = a - c - v_{IP} - b q_{-j} \tag{7}$$

By symmetry, the optimal decisions of the network producers are identical, and the same is valid for the independent producers. Hence, in the remainder of the paper we restrict our attention to solutions that satisfy this condition and denote by q_{NP}^* (q_{IP}^*) the production quantity of each network (independent) producer.

LEMMA 2. *The response functions for the network producers and the independent producers are as given below:*

$$q_{NP}^*(w, q_{IP}) = \begin{cases} \dfrac{f_{NP}(w, q_{IP})}{(n-m+1)\,\alpha b} & \textit{if } f_{NP}(w, q_{IP}) > 0 \\ 0 & \textit{otherwise} \end{cases} \tag{8}$$

$$q_{IP}^{*}(w,q_{NP}) = \begin{cases} \dfrac{f_{IP}(w,q_{NP})}{(m+1)\,b} & \textit{if } f_{IP}(w,q_{NP}) > 0 \\ 0 & \textit{otherwise} \end{cases} \tag{9}$$

where,

$$f_{NP}(w,q_{IP}) = (1-\alpha)\,w + \alpha\,(a - v_{NP}) - c - \alpha b m q_{IP} \tag{10}$$

$$f_{IP}(w,q_{NP}) = a - c - v_{IP} - b\,(n-m)\,q_{NP} \tag{11}$$

The response functions described in Lemma 2 are useful for determining the production strategies of network and independent producers in equilibrium for a given w and α. In Theorem 1 below we characterize the resulting equilibrium and show it is Nash equilibrium.

Theorem 1. *The unique Nash equilibrium of the producer's problem is given as follows:*

$$q_{NP}^{*}(w) = \begin{cases} 0 & \textit{if } w \le w_{\min} \\ \dfrac{k_{NP}(w)}{(n+1)\,\alpha b} & \textit{if } w_{\min} < w < w_{\max} \\ \dfrac{(1-\alpha)\,w + \alpha\,(a-v_{NP}) - c}{(n-m+1)\,\alpha b} & \textit{if } w \ge w_{\max} \end{cases} \tag{12}$$

$$q_{IP}^{*}(w) = \begin{cases} \dfrac{a-c-v_{IP}}{(m+1)\,b} & \textit{if } w \le w_{\min} \\ \dfrac{k_{IP}(w)}{(n+1)\,\alpha b} & \textit{if } w_{\min} < w < w_{\max} \\ 0 & \textit{if } w \ge w_{\max} \end{cases} \tag{13}$$

where,

$$w_{\min} = c - \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{(a-c-v_{NP}) + m\,(v_{IP}-v_{NP})}{(m+1)}\right] \tag{14}$$

$$w_{\max} = c + \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{(a-c-v_{IP}) - (n-m)\,(v_{IP}-v_{NP})}{(n-m)}\right] \tag{15}$$

$$k_{NP}(w) = (m+1)\,(1-\alpha)\,(w-c) + \alpha\,(a-c-v_{NP}) + m\alpha\,(v_{IP}-v_{NP}) \tag{16}$$

$$k_{IP}(w) = \alpha\,(a-c-v_{IP}) - (n-m)\,[(1-\alpha)\,(w-c) + \alpha\,(v_{IP}-v_{NP})] \tag{17}$$

Besides describing the equilibrium solution, Theorem 1 provides several interesting results. First, it indicates that there exists a threshold procurement price $w_{\min}$ below which the network producers do not compete. It may be noted that the threshold is lower than the network producer's marginal cost c, and takes into account the additional revenue that accrues due to surplus sharing mechanism. Similarly, the theorem indicates that there exists a threshold procurement price $w_{\max}$ above which the independent producers do not compete. Since $w_{\min} \le w_{\max}$, in the interval $[(w_{\min}),(w_{\max})]$, both network and independent producers compete in the market.

Propositions 1 and 2 below provide a summary of the description above and the insights from Theorem 1.

Proposition 1. *$w_{\min}$ is decreasing in both α and v_{IP}, and it is increasing in m. $w_{\max}$ is decreasing in v_{IP}, and it is increasing in both α and m.*

$w_{\min}$ decreasing in α shows that lower procurement price is sufficient for higher level of profit sharing so that the network producers compete. Similarly, lower procurement price suffices for higher v_{IP} so that the network producers compete. On the contrary, with increasing competition in the market, i.e., m, higher procurement price is required. Higher value of v_{IP} indicates that the independent producers are less competitive (in comparison with the network producers) and drop out of the market at lower value of w as reflected by decreasing $w_{\max}$. On the contrary, for higher profit sharing, i.e., α, and/or competition in the market, i.e., m, the independent producers drop out of the market at higher value of w as reflected by increasing $w_{\max}$.

Proposition 2. *In equilibrium, production quantity of the network producers is non-decreasing and that of the independent producers is non-increasing in the procurement price w.*

4.2. Marketing Agent's Problem

As mentioned earlier, the marketing agent is the Stackelberg leader for the producers in the market, and its problem is to maximize the network surplus, as given in (3). In this section, we assume that α is given and analyze the marketing agent's decision on procurement price w. (Choice of α is discussed in Section 5). By the assumptions of complete information and rationality of the players in the game, the marketing agent anticipates the response functions of the network producers and the independent producers, and the resulting Nash equilibrium as given in Theorem 1, and uses the information for choosing the procurement price w. The resulting profit function for the supply chain network is as given below:

$$\pi_N(w) = \begin{cases} 0 & \text{if } w \leq w_{\min} \\ \{a - c - v_{NP} - b[(n-m)\, q^*_{NP}(w) + m q^*_{IP}(w)]\}\,(n-m)\, q^*_{NP}(w) & \text{if } w_{\min} < w < w_{\max} \\ \{a - c - v_{NP} - b\,(n-m)\, q^*_{NP}(w)\}\,(n-m)\, q^*_{NP}(w) & \text{if } w \geq w_{\max} \end{cases} \tag{18}$$

where $q^*_{NP}(w)$ and $q^*_{IP}(w)$ are as given in Theorem 1.

The marketing agent's problem is to maximize $\pi_N(w)$ such that $w \geq 0$. First, we characterize the objective function as follows:

Lemma 3. *The objective function of the marketing agent's problem, i.e., $\pi_N(w)$, is strictly concave in w over the regions $[w_{\min}, w_{\max})$ and $(w_{\max}, +\infty)$.*

From Lemma 3, we observe that π_N is strictly concave in its argument over two disjoint regions with a possible non-conformity at $w_{\max}$. In Theorem 2 below, we show that the function is concave in its argument over the domain formed by the union of the two disjoint regions.

Theorem 2. *The objective function of the marketing agent's problem, i.e., $\pi_N(w)$, is strictly concave in w for $w \geq w_{\min}$.*

Corollary 1. *The marketing agent's problem of maximizing the network surplus is a convex program; the first order optimality conditions are necessary and sufficient.*

In Theorem 3 below we characterize the optimal solution for the marketing agent's problem.

Theorem 3. *The unique optimal solution for the marketing agent's problem is as given below:*

$$w^* = \begin{cases} w_1^* & \text{if } v_{NP} < v_{IP} < v_m^s \\ w_{\max} & \text{if } v_m^s \leq v_{IP} < v^s \\ w_2^* & \text{if } v^s \leq v_{IP} \leq (a-c) \end{cases} \tag{19}$$

where,

$$w_1^* = c - \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{(n-2m-1)\left[(a-c-v_{NP}) + m(v_{IP}-v_{NP})\right]}{2(n-m)(m+1)}\right] \tag{20}$$

$$w_2^* = c - \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{(n-m-1)(a-c-v_{NP})}{2(n-m)}\right] \tag{21}$$

$$v_m^s = v_{NP} + \frac{a-c-v_{NP}}{(m+2)} \tag{22}$$

$$v^s = v_{NP} + \frac{a-c-v_{NP}}{2} \tag{23}$$

Corollary 2. *A characterization of the optimal solution to the marketing agent's problem, MA, is given in Table 2.*

	$v_{NP} < v_{IP} < v_m^s$	$v_m^s \leq v_{IP} \leq v^s$	$v^s < v_{IP} \leq (a-c)$
w^*	w_1^*	$w_{\max}$	w_2^*
q_{NP}^*	$\frac{a-c-v_{NP}+m(v_{IP}-v_{NP})}{2(n-m)b}$	$\frac{a-c-v_{IP}}{(n-m)b}$	$\frac{a-c-v_{NP}}{2(n-m)b}$
q_{IP}^*	$\frac{a-c-v_{IP}-(m+1)(v_{IP}-v_{NP})}{2(m+1)b}$	0	0
p^*	$\frac{a+(2m+1)c+(m+1)v_{NP}+mv_{IP}}{2(m+1)}$	$c+v_{IP}$	$\frac{a+c+v_{NP}}{2}$
π_{NP}^*	$\alpha b(q_{NP}^*)^2$	$\alpha b(q_{NP}^*)^2$	$\alpha b(q_{NP}^*)^2$
π_{IP}^*	$b(q_{IP}^*)^2$	0	0
π_N^*	$\frac{b((n-m)q_{NP}^*)^2}{(m+1)}$	$\frac{1}{b}(v_{IP}-v_{NP})(a-c-v_{IP})$	$\frac{1}{b}\left(\frac{a-c-v_{NP}}{2}\right)^2$

Table 2 Optimal Solution to Problem MA

Table 2 provides several interesting insights and describes the impact of various factors on the equilibrium condition resulting from optimal decision on procurement price, w. First, while the independent producers can be shut out of the market when v_{IP} is relatively large (be appropriate choice of w), if is not surprising that network producers will always participate in the market. Second, when v_{IP} is relatively low both network and independent producers compete in the market. With increasing v_{IP}, the independent producers become less competitive and q^*_{NP} increases. This behavior holds when $v_{IP} < v^s_m$. The pattern of supply from the network producers (q^*_{NP}) change in the region $v^s_m \leq v_{IP} \leq v^s$. In this region, q^*_{NP} is decreasing in v_{IP}. Essentially, in this region the marketing agent's price w is such that the independent producers are shut out of the market. Beyond v^s, the independent producers are no longer competitive and marketing agent behaves like a monopolist. While the production quantity is not monotonic, the network surplus (and the surplus of the network producers) is monotonically increasing in v_{IP} in the region $v_{NP} < v_{IP} \leq v^s$ and constant beyond v^s, thus resulting lower competitiveness from the independent producers.

More interesting is the result that shows that network surplus depends on cost parameters and m, the number of independent producers, but is independent of α. The result be intuitively understood as α and w reflect internal mechanism among the network producers and that should not affect the network's competitive position vis-à-vis the independent producers. However, as we discuss in more detail in the next section, the choice of α is important since it has implications for network stability.

The results of Table 2 are also useful for assessing the impact of competition through m, the number of independent producers. It may be noted that competition for the network is diluted with increasing m and results in higher surplus and production from the network producers. The market price likewise is increasing in m and v_{IP} in the region in which both network and independent producers are active. We conclude this section with the observation that together these results are useful in understanding the implications of different factors and may be helpful developing strategies to compete effectively.

5. Network Stability

In this section we use the results of the previous section to examine issues related to the stability of the supply chain. In particular, our objective is to identify conditions under which the cooperative arrangement of the network producers is stable. Accordingly, we adopt the notion of pairwise stability introduced by Jackson and Wolinsky (1996) and define the network to be internally stable if there is no economic incentive for the network producers to break the network relationship and operate as independent producer, i.e., $\pi^*_{NP}(m) > \pi^*_{IP}(m+1)$. Similarly, we define the network to be externally stable if there is no incentive for the independent producers to join the network, i.e., $\pi^*_{IP}(m) > \pi^*_{NP}(m-1)$. Clearly, a network that is both internally and externally stable is stable with incentives for either player to alter the existing arrangement. External instability implies that the network is attractive and that the independent producers have an interest in becoming a part of the network. (In a completely different context, this kind of behavior is observable in sports issues such as NFL in the USA, IPL in India.) Internal instability, on the other hand, implies that the cooperative arrangement is not sustainable and likely to disintegrate in the absence of compliance mechanism. In the remainder of this section we examine these issues with focus on the impact of three factors: (i) cost parameters (v_{NP}, v_{IP}), (ii) number of producers m and level of competition, and (iii) profit sharing α.

5.1. Impact of Number of Producers

Proposition 3. *The network is internally stable and externally unstable if $m > m^s$, where*

$$m^s = \lfloor \frac{a - c - v_{IP}}{v_{IP} - v_{NP}} \rfloor - 1 \tag{24}$$

Proposition 3 provides a bound on the number of independent producers, or equivalently, the size of the network, such that the network is internally stable and externally unstable if the number of independent producers is more than the lower bound. In other words, if the competition among the producers in the market is beyond a threshold value, given by m^s, no network producer has any incentives to sever its linkages in the network and compete independently. On the contrary, every independent producer has incentives to join the network. From (24), we also observe that m_s is non-increasing in v_{IP} which suggests that internally stability of the network improves for the increasing marginal cost of the independent producers.

From Corollary 2, for $v_{NP} < v_{IP} < v^s_m$, we know that the net profit for a network producer a function of the value of α selected by the marketing agent. Therefore, whether the network is stable or not depends on α. Extending the results from Proposition 3, we can say that, for $m \le m_s^{v_{IP}}$, the stability of the network depends on α too.

Proposition 4. *If $(v_{IP} - v_{NP}) > (a - c - v_{NP})\left(\frac{1}{2} - \frac{\sqrt{\alpha}}{n}\right)$, the network is internally stable. Otherwise, there exists an m^s_l such that the network is internally stable for $m > m^s_l$ and internally unstable for $m < m^s_l$, where m^s_l is the smallest m that satisfies the following condition:*

$$\sqrt{\alpha}\left[\frac{(a - c - v_{NP}) + n(v_{IP} - v_{NP})}{n - m}\right] + \left(1 - \sqrt{\alpha}\right)(v_{IP} - v_{NP}) > \frac{a - c - v_{NP}}{m + 2} \tag{25}$$

Proposition 5. *If* $(a-c-v_{IP})\left(\dfrac{1}{n+1}-\sqrt{\alpha}\right) > (1+n\sqrt{\alpha})(v_{IP}-v_{NP})$*, the network is externally stable. Otherwise, there exists an* m_u^s *such that the network is externally unstable for* $m > m_u^s$ *and externally stable for* $m < m_u^s$*, where* m_u^s *is the smallest* m *which satisfies the following condition:*

$$\frac{a-c-v_{IP}}{m+1} > \sqrt{\alpha}\left[\frac{(a-c-v_{NP})+n(v_{IP}-v_{NP})}{n-m+1}\right] + (1-\sqrt{\alpha})(v_{IP}-v_{NP}) \tag{26}$$

Proposition 3, 4 and 5 describe some properties of network stability as a function of cost parameters (v_{IP}, v_{NP}), profit share (α), and the number of producers. For example, for given costs v_{IP}, v_{NP} and choice of α, Proposition 4 identifies a lower bound on the number of independent producers for internal stability of the network. If the number of independent producers is below the lower bound, the network is internally unstable and there exist incentives for some producers in the network to sever links with the marketing agent. Similarly, Proposition 5 identifies an upper bound on the number of independent producers for external stability. If the number of independent producers is above the upper bound, the network is externally unstable. Together Proposition 4 and 5 identify a range for the number of independent producers in which the network is stable. We note that m_l^s is non-increasing in v_{IP}. This is consistent with the intuition that as the independent producers become less competitive (higher v_{IP}), the network is internally stable. Similarly, we note that m_u^s is non-increasing in v_{IP}. The proposition also indicates that if the level of competition from the independent producers is known (m is given), choice of α and network efficiency (v_{NP}) may have an impact on network stability. These aspects are discussed in the following section.

Corollary 3. *If* $m_l^s \leq m_u^s$*, then there exists a range of* m*, given by* $[m_l^s, m_u^s]$*, in which the network is internally as well as externally stable.*

5.2. Impact of Marginal Cost

Proposition 6. *If* $v_{IP} \geq v_m^s$*, the network is internally stable but it is external unstable.*

Clearly, the relative cost of network and independent producers is a crucial parameter with implications for stability. Proposition 6 identifies the threshold value, $v_m^s - v_{NP} = \dfrac{a-c-v_{NP}}{m+2}$, for the cost difference beyond which independent produces exert no impact on the supply chain. Further it may be noted that this threshold cost difference is decreasing in m which suggests that with increase in the number of independent producers, the increased competition improves the network stability.

From Corollary 2, we know that the net profit for a network producer is a function of the value of α selected by the marketing agent. Therefore, whether the network is stable or not depends on

α too. Extending the results from Proposition 6, we derive results on network stability in relation to the marginal cost of the independent producers and profit sharing parameter α.

Theorem 4. *For given m and α, such that $v_{NP} < v_{IP} < v_m^s$, the network is internally stable if $v_{IP} > v_l^s$, and it is internally unstable if $v_{IP} < v_l^s$. Similarly, the network is externally stable if $v_{IP} < v_u^s$, and it is externally unstable if $v_{IP} > v_u^s$. Here,*

$$v_l^s = v_{NP} + \frac{\left[(n-m) - \sqrt{\alpha}\,(m+2)\right](a-c-v_{NP})}{(n-m)(m+3) + \sqrt{\alpha}m\,(m+2)} \tag{27}$$

$$v_u^s = v_{NP} + \frac{\left[(n-m+1) - \sqrt{\alpha}\,(m+1)\right](a-c-v_{NP})}{(n-m+1)(m+2) + \sqrt{\alpha}\,(m-1)(m+1)} \tag{28}$$

The network is both internally and externally stable if $v_l^s \leq v_{IP} \leq v_u^s$.

Theorem 4 states that there exist bounds on the marginal cost of sales and distribution of the product for the independent producers so that the network is internally and/or externally stable. By showing that $v_l^s \leq v_u^s \leq v_m^s$, we can conclude that the network stability is always guaranteed for a certain range of the marginal cost of sales and distribution of the product for the independent producers. Figure 2 shows the interrelationship between the network stability for the price and profit sharing based coordination mechanism and the marginal cost for the producers.

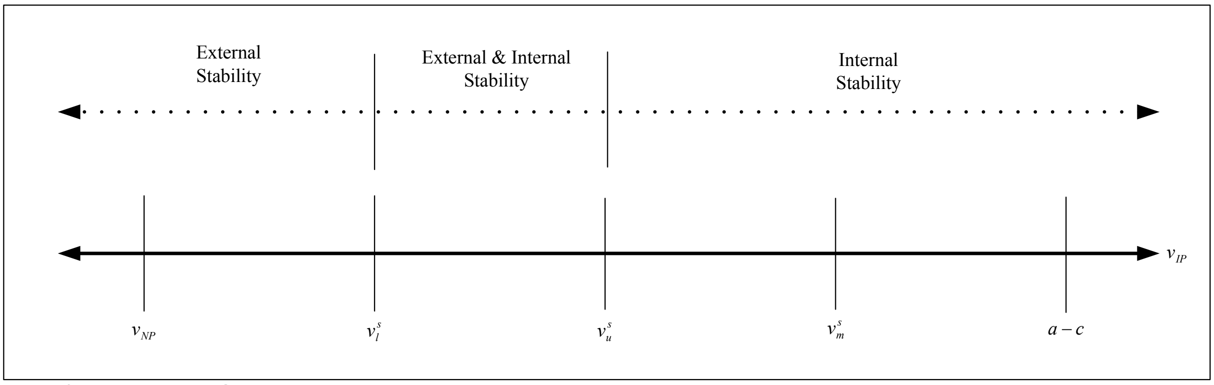

Figure 2 Network Stability - Cost Differences

From (27) and (28), we observe that both v_l^s and v_u^s are decreasing in α. Therefore, we say that, higher the profit share, lower are the bounds on the marginal cost of the independent producers so that the network is internally and externally stable. In other words, for higher level of profit sharing, the network is internally stable even for the *lower* marginal cost for the independent producers. A managerial insight from this observation is that, for given v_{IP} and m, the profit sharing parameter, α, can be used as a tool to induce stability in the network.

5.3. Impact of Profit Sharing

Recall that coordination mechanism for the network producers involves α, the profit sharing parameter. The analysis in Section 4 is based on the assumption that α is given, and $0 < \alpha < 1$. In this section we first complete the analysis by considering the special cases with $\alpha = 0$ and $\alpha = 1$ and discuss implication of the choice of α. While results in Section 4 suggest that network surplus is independent of α, the producers' share is influenced by α, and hence, impacts network stability. In our analysis, we use we use Lemma 2 that characterizes the response functions for both network and independent producers.

5.3.1. Special Case : $\boldsymbol{\alpha = 0}$

For $\alpha = 0$, we note the price and profit sharing based coordination mechanism reduces to price only mechanism. When $w > c$, we observe that $q^*_{NP} = \infty$, $q^*_{IP} = 0$, and hence, $\pi^*_N = -\infty$. Therefore, $w > c$ is not an optimal solution for the marketing agent's problem. When $w < c$, $q^*_{NP} = 0$, $q^*_{IP} = (a - c - v_{IP}) / (m+1) b$, and hence, $\pi^*_N = 0$ and $\pi^*_{IP} = \left[(a - c - v_{IP}) / (m+1)\right]^2 b \geq 0$. Accordingly, for $w < c$, the network producers do not compete in the market. When $w = c$, the system is indeterminate and equilibrium is not assured. When $\alpha = 0$, $\pi^*_N = (-\infty, 0]$, and hence, $\alpha = 0$ is not a solution for the marketing agent's problem.

5.3.2. Special Case : $\boldsymbol{\alpha = 1}$

The analysis of this case parallels that in Section 4. When $\alpha = 1$, Lemma 2 can be rewritten as follows:

Lemma 4. *The response functions for the network producers and the independent producers are as given below:*

$$q^*_{NP}(q_{IP}) = \begin{cases} \dfrac{f_{NP}(q_{IP})}{(n-m+1)\,b} & \textit{if } f_{NP}(q_{IP}) > 0 \\ 0 & \textit{otherwise} \end{cases} \tag{29}$$

$$q^*_{IP}(q_{NP}) = \begin{cases} \dfrac{f_{IP}(q_{NP})}{(m+1)\,b} & \textit{if } f_{IP}(q_{NP}) > 0 \\ 0 & \textit{otherwise} \end{cases} \tag{30}$$

where,

$$f_{NP}(q_{IP}) = a - c - v_{NP} - bmq_{IP} \tag{31}$$

$$f_{IP}(q_{NP}) = a - c - v_{IP} - b(n-m)\,q_{NP} \tag{32}$$

Here, we note that the response functions for both independent and network producers are independent of the procurement price w. This is due to the fact that with $\alpha = 1$, the entire profit of the network is shared with the network producers, and hence, the *intermediate* procurement price from the marketing agent is inconsequential for the producers while determining their supply quantity. Theorem 5 below provides a complete characterization of the resulting equilibrium.

THEOREM 5. *For $\alpha = 1$, the Nash equilibrium of the producer's problem is given in Table 3.*

	$v_{NP} < v_{IP} < v^s_{m\|\alpha=1}$	$v^s_{m\|\alpha=1} \leq v_{IP} \leq (a-c)$
q^*_{NP}	$\dfrac{(a-c-v_{NP})+m\,(v_{IP}-v_{NP})}{(n+1)\,b}$	$\dfrac{a-c-v_{NP}}{(n-m+1)\,b}$
q^*_{IP}	$\dfrac{(a-c-v_{IP})-(n-m)\,(v_{IP}-v_{NP})}{(n+1)\,b}$	0
p^*	$\dfrac{a+n\,(c+v_{NP})+m\,(v_{IP}-v_{NP})}{(n+1)}$	$\dfrac{a+(n-m)\,(c+v_{NP})}{(n-m+1)}$
π^*_{NP}	$b\,(q^*_{NP})^2$	$b\,(q^*_{NP})^2$
π^*_{IP}	$b\,(q^*_{IP})^2$	0
π^*_{N}	$(n-m)\,b\,(q^*_{NP})^2$	$(n-m)\,b\,(q^*_{NP})^2$

Table 3 Nash Equilibrium when $\alpha = 1$

$$\textit{where,} \qquad v^s_{m|\alpha=1} = v_{NP} + \frac{a-c-v_{NP}}{(n-m+1)} \tag{33}$$

PROPOSITION 7. *The optimal solution for the marketing agent's problem is such that $\alpha < 1$.*

Proposition 7 shows that the optimal strategy for the marketing agent is not share the entire network surplus among the network producers. From Theorem 5 above, for $\alpha = 1$, we also note that the equilibrium profit for the network producers is never zero, and it is a non-decreasing in v_{IP} and increasing in m. The bound on v_{IP} given by $v^s_{m|\alpha=1}$ has the same characteristics as that of v^s_m observed in Section 4. The network is internally stable and externally unstable, for given m, if $v_{IP} > v^s_{m|\alpha=1}$.

PROPOSITION 8. *Define*

$$\alpha^s_l = \left[\frac{q^*_{IP}\,(m+1)}{q^*_{NP}\,(m)}\right]^2, \qquad \textit{and} \qquad \alpha^s_u = \left[\frac{q^*_{IP}\,(m)}{q^*_{NP}\,(m-1)}\right]^2 \tag{34}$$

The network is internally stable if $\alpha > \alpha^s_l$, and it is internally unstable if $\alpha < \alpha^s_l$. Also, the network is externally stable if $\alpha < \alpha^s_u$, and it is externally unstable if $\alpha > \alpha^s_u$.

COROLLARY 4. *$\alpha^s_l \leq \alpha^s_u$, and hence, the network is internally as well as externally stable for $\alpha \in (\alpha^s_l, \alpha^s_u)$. (Refer Figure 3)*

Proposition 8 and Corollary 4 together bring out the influence of α on network stability. Proposition 8 identifies the threshold level of α, (α_l^s) that is needed to keep the network stable. it is interesting to note that α_l^s is non-increasing in m thus showing that the network internally stable at lower level of α with more competition from the independent producers. (α_l^s, α_u^s) denotes the range of α in which both network and independent producers compete. Since the network surplus is independent of α, the choice of α may be guided by stability conditions and the desire of the marketing agent and the network producers to maintain advantage over the independent producers.

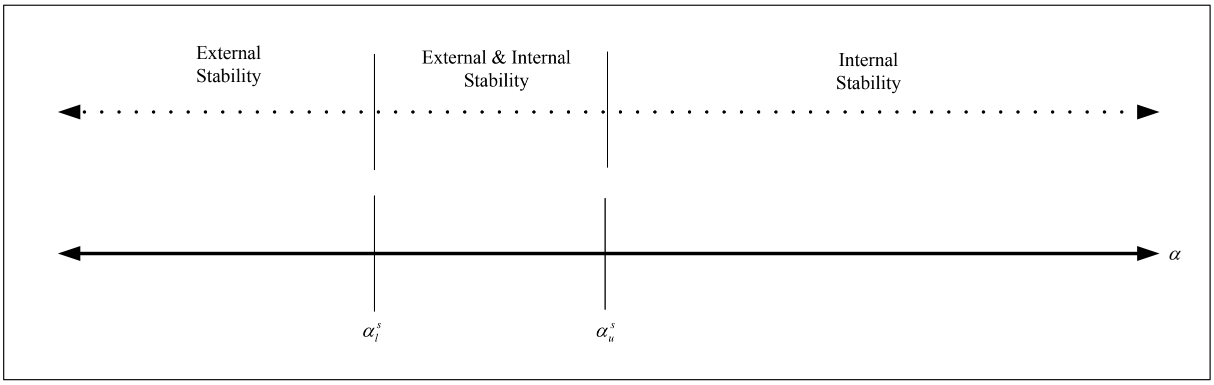

Figure 3 **Network Stability - Profit Sharing**

6. Conclusions

In this paper we have developed a game-theoretic model to study stability issues in two-tier supply chain comprising to two types of producers - independent producers who compete directly in the market and network producers that compete through a marketing agent who acts as a coordinator. The model is appropriate for supply chains with exclusivity contracts, franchises, cooperatives, etc. that operate in a competitive environment. While coordination issues have received much attention in the literature with focus on performance/efficiency and design and analysis of coordination mechanisms, stability issues have largely remained unexplored. The work reported in this paper is complementary in nature and bridges this gap in the literature.

Our results can be broadly classified into two categories: (i) Characterizing the equilibrium solutions resulting from the parameter choices of the coordination mechanism and describing the impact of cost parameters and the number of independent producers on the network surplus, prices, production, etc. (ii) Network stability: Here we identify conditions under which the network is stable and develop bounds on parameters for assuring stability. Our results are based on simplifying assumptions, nevertheless they are useful and provide insights and guidelines for managerial decision making. Of particular interest is the role of α, the profit sharing parameter. While the choice of α does not impact the network surplus, it has implications for network stability.

The model and the results presented in this paper may be interpreted as a building block in the development of a richer and a more comprehensive framework. We conclude the paper with some guidelines in this regard. First, our analysis is based on one coordination mechanism. While the one analyzed in the paper is common and popular, it may be useful to extend the analysis to other mechanisms and to the extent possible, generalize the results. Second, we ignored capacity constraints and assumed that the producers are identical. Extensions of the model to relax these assumptions may be useful in developing results that go beyond qualitative insights and provide aid for decision support.

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Appendix

Proof of Lemma 1. Since the network producer's problem is a convex program, the Kurush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimality. The KKT optimality conditions for network producer i's problem are as follows:

$$q_i^*(w, q_{-i}) = \frac{(1-\alpha)w + \alpha(a - v_{NP}) - c - \alpha b q_{-i} + \lambda_i^*}{2\alpha b} = \frac{f_i(w, q_{-i}) + \lambda_i^*}{2\alpha b} \tag{35}$$

$$q_i^* \lambda_i^* = 0 \tag{36}$$

$$q_i^*, \lambda_i^* \geq 0 \tag{37}$$

Clearly, q_i^* as defined in (4) together with $\lambda_i^* = \max\{0, -f_i(w, q_{-i})\}$ satisfies KKT conditions (35)-(37) and the Lemma follows. The second part of the Lemma characterizing response functions for the independent producers can be shown similarly. □

Proof of Lemma 2. Substitute $q_i^* = q_{NP}^*$, $i = 1, 2, ..., (n-m)$, and $q_j^* = q_{IP}^*$, $j = 1, 2, ..., m$, in (6)-(7), and simplify to obtain the results in Lemma 2. □

Proof of Theorem 1. To prove the theorem it is sufficient to show that the production quantities specified by (12) and (13) satisfy simultaneously the response functions (8) and (9) of Lemma 2. Since this solution denotes best response of each producer to the other's strategy, it follows from Osborne (2003) that the resulting equilibrium is a Nash equilibrium. Further, since the profit functions for network and independent producers are concave in their respective arguments, by Theorem 1.2 of Fudenberg and Tirole (1991), a pure strategy Nash equilibrium exists.

Note that k_{NP} is linear in w and $k_{NP}(w_{\min}) = 0$. k_{IP} is likewise linear in w with $k_{NP}(w_{\max}) = 0$. It follows that the response functions $q^*_{NP}(w)$ and $q^*_{IP}(w)$ are such that $q^*_{NP}(w) = 0\,(q^*_{IP}(w) = 0)$for $w \leq w_{\min}\,(w \geq w_{\max})$. We also note that $w_{\min} \leq w_{\max}$ from the fact that:

$$
\begin{aligned}
w_{\max} - w_{\min} &= \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{(a-c-v_{IP}) - (n-m)(v_{IP}-v_{NP})}{(n-m)} + \frac{(a-c-v_{NP}) + m(v_{IP}-v_{NP})}{(m+1)}\right] \\
&= \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{(n+1)(a-c-v_{IP})}{(n-m)(m+1)}\right] > 0 \qquad \text{[By Assumption 1]}
\end{aligned}
$$

Case 1. $w \leq w_{\min}$

Clearly, $k_{NP} \leq 0$, and hence, $q^*_{NP}(w) = 0$ satisfies (8). Also, q^*_{IP} given by (13), satisfies (9).

Case 2. $w_{\min} < w < w_{\max}$

$k_{IP} > 0$ and $q^*_{IP}(w)$ given by (13), from (10), we obtain:

$$
\begin{aligned}
f_{NP} &= (1-\alpha)w + \alpha(a - v_{NP}) - c - \alpha bm\frac{k_{IP}(w)}{(n+1)\alpha b} \\
&= (1-\alpha)w + \alpha(a - v_{NP}) - c - \alpha bm\left\{\frac{\alpha(a-c-v_{IP}) - (n-m)[(1-\alpha)(w-c) + \alpha(v_{IP}-v_{NP})]}{(n+1)\alpha b}\right\} \\
&= \frac{(n-m+1)[(m+1)(1-\alpha)(w-c) + \alpha(a-c-v_{NP}) + m\alpha(v_{IP}-v_{NP})]}{(n+1)} \\
&= \frac{(n-m+1)k_{NP}}{(n+1)} > 0
\end{aligned}
$$

Hence, $q^*_{NP}(w)$ given by (12) satisfies (8). Similarly, $k_{NP} > 0$ and $q^*_{NP}(w)$ given by (12), from (11), we obtain:

$$
\begin{aligned}
f_{IP} &= a - c - v_{IP} - b(n-m)\frac{k_{NP}(w)}{(n+1)\alpha b} \\
&= a - c - v_{IP} - b(n-m)\left\{\frac{(m+1)(1-\alpha)(w-c) + \alpha(a-c-v_{NP}) + m\alpha(v_{IP}-v_{NP})}{(n+1)\alpha b}\right\} \\
&= \frac{(m+1)[\alpha(a-c-v_{IP}) - (n-m)[(1-\alpha)(w-c) + \alpha(v_{IP}-v_{NP})]]}{(n+1)} \\
&= \frac{(m+1)k_{IP}}{(n+1)} > 0
\end{aligned}
$$

Hence, $q^*_{IP}(w)$ given by (13) satisfies (9).

Case 3. $w \geq w_{\max}$

This case is similar to Case 1 above. $k_{IP} \leq 0$, and hence, $q^*_{IP}(w) = 0$ satisfies (9). Also, q^*_{NP} given by (12), satisfies (8). ☐

Proof of Proposition 1. The proof of $w_{\min}$ decreasing in both α and v_{IP} is straightforward from (14).

$$
\begin{aligned}
&w_{\min}(m+1) - w_{\min}(m) \\
&= -\left(\frac{\alpha}{1-\alpha}\right)\left\{\left[\frac{(a-c-v_{NP}) + (m+1)(v_{IP}-v_{NP})}{(m+2)}\right] - \left[\frac{(a-c-v_{NP}) + m(v_{IP}-v_{NP})}{(m+1)}\right]\right\} \\
&= \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{(a-c-v_{IP})}{(m+1)(m+2)}\right] > 0 \qquad \text{[By Assumption 1]}
\end{aligned}
$$

Hence, $w_{\min}$ is increasing in m. The proof of $w_{\max}$ decreasing in v_{IP} and increasing in both α and m is straightforward from (15). ☐

Proof of Proposition 2. From (12), $\dfrac{dq^*_{NP}}{dw} = 0$ when $w \leq w_{\min}$. Also, $\dfrac{dq^*_{NP}}{dw} = \dfrac{(m+1)(1-\alpha)}{(n+1)\alpha b} > 0$ when $w_{\min} < w < w_{\max}$, and $\dfrac{dq^*_{NP}}{dw} = \dfrac{(1-\alpha)}{(n-m+1)\alpha b} > 0$ for $w \geq w_{\max}$. Therefore, $\dfrac{dq^*_{NP}}{dw} \geq 0$. Similarly, we can prove that $\dfrac{dq^*_{IP}}{dw} \leq 0$. ☐

Proof of Lemma 3. For $w_{\min} < w < w_{\max}$,

$$\begin{aligned}
\pi_N^{'} &= \left[\frac{(n-m)(1-\alpha)}{(n+1)^2\alpha^2 b}\right]\Big\{-2(n-m)(m+1)(1-\alpha)(w-c) \\
&\quad -(n-2m-1)\alpha\left[(a-c-v_{NP})+m(v_{IP}-v_{NP})\right]\Big\} \\
\pi_N^{''} &= \left[\frac{(n-m)(1-\alpha)}{(n+1)^2\alpha^2 b}\right]\Big\{-2(n-m)(m+1)(1-\alpha)\Big\} < 0
\end{aligned} \tag{38}$$

where, $\pi_N^{'} = \dfrac{d\pi_N}{dw}$ and $\pi_N^{''} = \dfrac{d^2\pi_N}{dw^2}$. Similarly, for $w > w_{\max}$,

$$\begin{aligned}
\pi_N^{'} &= \left[\frac{(n-m)(1-\alpha)}{(n-m+1)^2\alpha^2 b}\right]\Big\{-2(n-m)(1-\alpha)(w-c)-(n-m-1)\alpha(a-c-v_{NP})\Big\} \\
\pi_N^{''} &= \left[\frac{(n-m)(1-\alpha)}{(n+1)^2\alpha^2 b}\right]\Big\{-2(n-m)(1-\alpha)\Big\} < 0
\end{aligned} \tag{39}$$

Therefore, π_N is strictly concave in w over $[w_{\min}, w_{\max})$ and $(w_{\max}, +\infty)$. □

Proof of Theorem 2. First, we show that π_N is continuous in the interval $w \geq w_{\min}$ and differentiable as well, except at $w = w_{\max}$.

Note that for $w \geq w_{\max}$, from Theorem 1 it follows that

$$q_{IP}^{*} = 0; \quad q_{NP}^{*} = \frac{(1-\alpha)w_{\max}+\alpha(a-v_{NP})-c}{(n-m+1)\alpha b} = \frac{a-c-v_{IP}}{(n-m)b}$$

Therefore,

$$\begin{aligned}
\pi_N\left(w_{\max}^{+}\right) &= \left[a-c-v_{NP}-b(n-m)\frac{a-c-v_{IP}}{(n-m)b}\right](n-m)\frac{a-c-v_{IP}}{(n-m)b} \\
&= \frac{1}{b}(v_{IP}-v_{NP})(a-c-v_{IP}) \\
\pi_N^{'}\left(w_{\max}^{+}\right) &= \left[\frac{(n-m)(1-\alpha)}{(n-m+1)\alpha b}\right]\Big\{-(a-c-v_{IP})+(v_{IP}-v_{NP})\Big\}
\end{aligned} \tag{40}$$

Also, for $w = w_{\max}^{-}$, i.e., $w \to w_{\max}^{-}$,

$$\begin{aligned}
k_{IP} &= 0, \text{ and hence, } q_{IP}^{*} = 0 \\
k_{NP} &= (m+1)(1-\alpha)(w_{\max}-c)+\alpha(a-c-v_{NP})+m\alpha(v_{IP}-v_{NP}) \\
&= (n+1)\alpha\left[\frac{(a-c-v_{IP})}{(n-m)}\right] \text{ and hence, } q_{NP}^{*} = \frac{a-c-v_{IP}}{(n-m)b}
\end{aligned}$$

Therefore,

$$\begin{aligned}
\pi_N\left(w_{\max}^{-}\right) &= \left[a-c-v_{NP}-b(n-m)\frac{a-c-v_{IP}}{(n-m)b}\right](n-m)\frac{a-c-v_{IP}}{(n-m)b} \\
&= \frac{1}{b}(v_{IP}-v_{NP})(a-c-v_{IP}) \\
\pi_N^{'}\left(w_{\max}^{-}\right) &= \left[\frac{(n-m)(1-\alpha)}{(n+1)\alpha b}\right]\Big\{-(a-c-v_{IP})+(m+1)(v_{IP}-v_{NP})\Big\}
\end{aligned} \tag{41}$$

From (40) and (41) it follows that $\pi_N(w,\alpha)$ is continuous but not differentiable at $w = w_{\max}$.

Second, we show that $\pi_N^{'}\left(w_{\max}^{-}\right) \geq \pi_N^{'}\left(w_{\max}^{+}\right)$. From Assumption 1 and Assumption 3 it follows that

$$\begin{aligned}
&\pi_N^{'}\left(w_{\max}^{-}\right)-\pi_N^{'}\left(w_{\max}^{+}\right) \\
&= \left[\frac{(n-m)(1-\alpha)}{\alpha b}\right]\left[\frac{-(a-c-v_{IP})+(m+1)(v_{IP}-v_{NP})}{(n+1)}-\frac{-(a-c-v_{IP})+(v_{IP}-v_{NP})}{(n-m+1)}\right]
\end{aligned}$$

$$= \left[\frac{m(n-m)(1-\alpha)}{\alpha b}\right]\left[\frac{(a-c-v_{IP})+(n-m)(v_{IP}-v_{NP})}{(n+1)(n-m+1)}\right] \geq 0 \tag{42}$$

From Lemma 3 and (42) above, it follows that

$$\pi_N^{'}(w_x) \geq \pi_N^{'}(w_{\max}^{-}) \geq \pi_N^{'}\left(w_{\max}^{+}\right) \geq \pi_N^{'}(w_y), \forall w_x \leq w_{\max} \leq w_y \tag{43}$$

From Lemma 3, it is straightforward that π_N is concave in w when $w < w_{\max}(\alpha)$ and when $w \geq w_{\max}(\alpha)$. Now, for $w_a, w_b \geq w_{\min}$, consider the following two cases:

Case 1. $w_a \leq w_{\max}(\alpha) \leq w_b$

$$\begin{aligned}
\pi_N(w_b) &\leq \pi_N(w_{\max}) + \pi_N^{'}\left(w_{\max}^{+}\right)(w_b - w_{\max}) && \text{[By Lemma 3]}\\
&\leq \pi_N(w_a) + \pi_N^{'}(w_a)(w_{\max} - w_a) + \pi_N^{'}\left(w_{\max}^{+}\right)(w_a - w_{\max}) && \text{[By Lemma 3]}\\
&\leq \pi_N(w_a) + \pi_N^{'}(w_a)(w_{\max} - w_a) + \pi_N^{'}(w_a)(w_a - w_{\max}) && \text{[By (43)]}\\
&\leq \pi_N(w_a) + \pi_N^{'}(w_a)(w_b - w_a)
\end{aligned}$$

Case 2. $w_b \leq w_{\max}(\alpha) \leq w_a$

The proof is similar to the Case 1 above.

Therefore, by Shapiro (1979), Bertsekas (1999), π_N is concave in w for $w \geq w_{\min}$. □

Proof of Theorem 3. First, we show that $v_m^s \leq v^s \leq (a-c)$. It follows from (22)-(23) that $v_m^s \leq v^s$. Further, $a - c - v^s = (a - c - v_{NP})/2 \geq 0$ by Assumption 2, and hence, $v_m^s \leq v^s \leq (a-c)$.

Case 1. $v_{NP} < v_{IP} < v_m^s$

Consider w_1^* as a solution for the marketing agent's problem. The feasibility of the solution, i.e., $w_{\min} < w_1^* < w_{\max}$ when $v_{IP} < v_m^s$ can be shown as follows:

$$\begin{aligned}
w_1^* - w_{\min} &= -\left(\frac{\alpha}{1-\alpha}\right)\left\{\left[\frac{(n-2m-1)\left[(a-c-v_{NP})+m(v_{IP}-v_{NP})\right]}{2(n-m)(m+1)}\right]\right.\\
&\qquad\left. - \left[\frac{(a-c-v_{NP})+m(v_{IP}-v_{NP})}{(m+1)}\right]\right\}\\
&= \left(\frac{\alpha}{1-\alpha}\right)\left[\frac{(n+1)\left[(a-c-v_{NP})+m(v_{IP}-v_{NP})\right]}{2(n-m)(m+1)}\right]\\
&\geq 0 \qquad \text{[By Assumption 1 and 3]}\\
w_{\max} - w_1^* &= \left(\frac{\alpha}{1-\alpha}\right)\left\{\left[\frac{(a-c-v_{IP})-(n-m)(v_{IP}-v_{NP})}{(n-m)}\right]\right.\\
&\qquad\left. + \left[\frac{(n-2m-1)\left[(a-c-v_{NP})+m(v_{IP}-v_{NP})\right]}{2(n-m)(m+1)}\right]\right\}\\
&= \left(\frac{\alpha}{1-\alpha}\right)\left\{\frac{(n+1)\left[(a-c-v_{NP})-(m+2)(v_{IP}-v_{NP})\right]}{2(n-m)(m+1)}\right\}\\
&> 0 \text{ if } v_{IP} < v_m^s
\end{aligned}$$

Also, from (38), w_1^* satisfies the first order optimality condition as shown below:

$$\begin{aligned}
\pi_N^{'}(w_1^*) &= \left[\frac{(n-m)(1-\alpha)}{(n+1)^2\alpha^2 b}\right]\left\{-2(n-m)(m+1)(1-\alpha)(w_1^*-c)\right.\\
&\qquad\left. -(n-2m-1)\alpha\left[(a-c-v_{NP})+m(v_{IP}-v_{NP})\right]\right\}\\
&= 0
\end{aligned}$$

Hence, w_1^* is the optimal solution.

Case 2. $v_{IP} > v^s$

Consider w_2^* as a solution for the marketing agent's problem, and it is feasible, i.e., $w_2^* \geq w_{\max}$ if $v_{IP} \geq v_m^s$ as shown below:

$$\begin{aligned}
&w_2^* - w_{\max} \\
&= -\left(\frac{\alpha}{1-\alpha}\right)\left\{\left[\frac{(n-m-1)(a-c-v_{NP})}{2(n-m)}\right] - \left[\frac{(a-c-v_{IP})-(n-m)(v_{IP}-v_{NP})}{(n-m)}\right]\right\} \\
&= -\left(\frac{\alpha}{1-\alpha}\right)\left\{\frac{(n-m+1)\left[(a-c-v_{NP})-2(v_{IP}-v_{NP})\right]}{2(n-m)}\right\} \\
&\geq 0 \text{ if } v_{IP} \geq v^s
\end{aligned}$$

Also, from (39), w_2^* satisfies the first order optimality condition as shown below:

$$\begin{aligned}
\pi_N^{'}(w_2^*) &= \left[\frac{(n-m)(1-\alpha)}{(n-m+1)^2\alpha^2 b}\right]\left\{-2(n-m)(1-\alpha)(w_2^*-c)-(n-m-1)\alpha(a-c-v_{NP})\right\} \\
&= 0
\end{aligned}$$

Hence, w_2^* is the optimal solution.

Case 3. $v_m^s \leq v_{IP} < v^s$

We prove that $w_{\max}$ is the optimal solution for the marketing agent's problem by showing that $\pi_N(w_{\max}) \geq \pi_N(w)$, $\forall w \geq w_{\min}$. Note that $w_1^* \geq w_{\max}$ and $w_2^* < w_{\max}$ for $v_m^s \leq v_{IP} < v^s$.

Consider any w_a such that $w_{\min} < w_a < w_{\max}$. By Lemma 3, we note that $\pi_N^{'}(w_{\max}^-) \geq 0$ as $\pi_N^{'}(w_1^*) = 0$. Therefore,

$$\begin{aligned}
\pi_N(w_a) &\leq \pi_N(w_{\max}) + \pi_N^{'}(w_{\max}^-)(w_a - w_{\max}) && \text{[By Lemma 3]} \\
&\leq \pi_N(w_{\max}) && \text{[Since } w_a < w_{\max}\text{]}
\end{aligned}$$

Similarly, consider any w_b such that $w_b \geq w_{\max}$. By Lemma 3, we note that $\pi_N^{'}(w_{\max}^+) \leq 0$ as $\pi_N^{'}(w_2^*) = 0$. Therefore,

$$\begin{aligned}
\pi_N(w_b) &\leq \pi_N(w_{\max}) + \pi_N^{'}(w_{\max}^-)(w_b - w_{\max}) && \text{[By Lemma 3]} \\
&\leq \pi_N(w_{\max}) && \left[\text{Since } \pi_N^{'}\left(w_{\max}^+\right) \leq 0\right]
\end{aligned}$$

Hence, when $v_m^s \leq v_{IP} < v^s$, $w_{\max}$ is the optimal solution. □

Proof of Corollary 2. The optimal solution to Problem MA is obtained by substituting $w^*(\alpha)$ from Theorem 3 in the expressions from Theorem 1. □

Proof of Proposition 3 From definition of m^s, for $m > m^s$, we have $m+2 \geq \dfrac{a-c-v_{IP}}{v_{IP}-v_{NP}} + 1 = \dfrac{a-c-v_{NP}}{v_{IP}-v_{NP}}$, and $v_m^s = v_{NP} + \dfrac{a-c-v_{NP}}{m+2} \leq v_{IP}$. From Corollary 2, for $v_{IP} \geq v_m^s$, $\pi_{IP}^*(m) = 0$ and $\pi_{NP}^*(m) > 0$. Further, since v_m^s is monotonically decreasing in m, it follows that $\pi_{IP}^*(m+1) = 0$ and $\pi_{NP}^*(m+1) > 0$, and hence, network is internally stable and externally unstable. □

Proof of Proposition 4. From Corollary 2, the condition for internal stability, $\pi_{NP}^*(m) > \pi_{IP}^*(m+1)$, is given by the following:

$$\sqrt{\alpha}\left[\frac{(a-c-v_{NP})+n(v_{IP}-v_{NP})}{n-m}\right] + \left(1-\sqrt{\alpha}\right)(v_{IP}-v_{NP}) > \frac{a-c-v_{NP}}{m+2}$$

1. It may be noted that LHS of (25) is increasing in m, while RHS is decreasing in m. Hence, if (25) is satisfied for $m=0$, it is also valid for all $m>0$ and the network is internally stable.

2. If $(v_{IP}-v_{NP})>(a-c-v_{NP})\left(\frac{1}{2}-\frac{\sqrt{\alpha}}{n}\right)$, (25) satisfies for $m=0$, and the first part of the proposition follows.

3. (25) is clearly satisfied for $m=n$.

4. If the condition of the proposition is not valid, i.e., $(v_{IP}-v_{NP})\leq(a-c-v_{NP})\left(\frac{1}{2}-\frac{\sqrt{\alpha}}{n}\right)$, then (1) and (3) above imply that (25) is satisfied for some integer m such that $0\leq m\leq n$. Let m_l^s denote the smallest integer that satisfies (25).

It follows that (25) is satisfied for all $m\geq m_l^s$ and the second part of the proposition follows. □

Proof of Proposition 5. From Corollary 2, the condition for external stability, $\pi_{IP}^*(m)>\pi_{NP}^*(m-1)$, is given by the following:

$$\frac{a-c-v_{IP}}{m+1}>\sqrt{\alpha}\left[\frac{(a-c-v_{NP})+n(v_{IP}-v_{NP})}{n-m+1}\right]+\left(1-\sqrt{\alpha}\right)(v_{IP}-v_{NP})$$

1. It may be noted that LHS of (26) is decreasing in m, while RHS is increasing in m. Hence, if (25) is satisfied for $m=n$, it is also valid for all $m<n$ and the network is externally stable.

2. If $(a-c-v_{IP})\left(\frac{1}{n+1}-\sqrt{\alpha}\right)>(1+n\sqrt{\alpha})(v_{IP}-v_{NP})$, (26) satisfies for $m=n$, and the first part of the proposition follows.

3. If the condition of the proposition is not valid, i.e., $(a-c-v_{IP})\left(\frac{1}{n+1}-\sqrt{\alpha}\right)\leq(1+n\sqrt{\alpha})(v_{IP}-v_{NP})$, then (1) above implies that (26) is satisfied for some integer m such that $m\leq n$. Let m_u^s denote the smallest integer that satisfies (26).

It follows that (26) is satisfied for all $m\leq m_u^s$ and the second part of the proposition follows. □

Proof of Proposition 6. From Corollary 2, for $v_{IP}\geq v_m^s$, $\pi_{IP}^*(m)=0$ and $\pi_{NP}^*(m)>0$. Further, since v_m^s is monotonically decreasing in m, it follows that $\pi_{IP}^*(m+1)=0$ and $\pi_{NP}^*(m+1)>0$, and hence, network is internally stable and externally unstable. □

Proof of Theorem 4. From Corollary 2, the condition for internal stability of the network reduces to $v_{IP}>v_l^s$, and the condition for external stability of the network translates into $v_{IP}>v_u^s$. Now, we show that $v_l^s\leq v_u^s$ as follows:

$$\begin{aligned}v_u^s-v_l^s&=\frac{\left[(n-m+1)-\sqrt{\alpha}(m+1)\right](a-c-v_{NP})}{(n-m+1)(m+2)+\sqrt{\alpha}(m-1)(m+1)}-\frac{\left[(n-m)-\sqrt{\alpha}(m+2)\right](a-c-v_{NP})}{(n-m)(m+3)+\sqrt{\alpha}m(m+2)}\\&=\frac{\left[(n-m)(n-m+1)+\sqrt{\alpha}(m+1)\left(2n+4-\sqrt{\alpha}(m+2)\right)\right](a-c-v_{NP})}{\left[(n-m+1)(m+2)+\sqrt{\alpha}(m-1)(m+1)\right]\left[(n-m)(m+3)+\sqrt{\alpha}m(m+2)\right]}\geq 0\end{aligned}$$

Also, from (28), we observe that v_u^s is decreasing in α, and for $\alpha=0$, $v_u^s=v_m^s$. Hence, $v_u^s\leq v_m^s$ is valid for $\alpha\in(0,1)$. To summarize, $v_l^s\leq v_u^s\leq v_m^s$. □

Proof of Lemma 4. Substitute $\alpha=1$ in Lemma 2 to get (29) – (32) □

Proof of Theorem 5. To prove the theorem it is sufficient to show that the production quantities specified in Table 3 satisfy simultaneously the response functions (29) and (30) of Lemma 4.

Case 1. $v_{IP}<v_{m|\alpha=1}^s$

For $q_{IP}^{*}=[(a-c-v_{IP})-(n-m)(v_{IP}-v_{NP})]/(n+1)b$, we obtain:

$$f_{NP}=(n-m+1)[(a-c-v_{NP})+m(v_{IP}-v_{NP})]/(n+1)\geq 0,\ \text{and hence,}$$
$$q_{NP}^{*}=[(a-c-v_{NP})+m(v_{IP}-v_{NP})]/(n+1)b \text{ which satisfies (29).}$$

Similarly, for $q_{NP}^{*}=[(a-c-v_{NP})+m(v_{IP}-v_{NP})]/[(n+1)b]$, we obtain:

$$f_{IP}=(m+1)[(a-c-v_{IP})-(n-m)(v_{IP}-v_{NP})]/(n+1),\ \text{and hence,}$$
$$q_{IP}^{*}=[(a-c-v_{IP})-(n-m)(v_{IP}-v_{NP})]/(n+1)b \text{ which satisfies (30).}$$

Case 2. $v_{IP}\geq v^{s}_{m|\alpha=1}$

This case is similar to Case 1 above, and hence, proof is omitted.

It is straightforward to obtain equilibrium price and profit for the producers. □

Proof of Proposition 7. We prove the proposition by comparing $\pi_N^*(\alpha<1)$ from Corollary 2 with $\pi_N^*(\alpha=1)$ from Theorem 5 above.

Case 1. $m\leq\dfrac{n-1}{2}$

From (22) and (33), we note that:

$$v^{s}_{m|\alpha=1}-v^{s}_{m}=\frac{a-c-v_{NP}}{n-m+1}-\frac{a-c-v_{NP}}{m+2}=\frac{(a-c-v_{NP})(2m-n+1)}{(n-m+1)(m+2)}\leq 0$$

Therefore, $v^{s}_{m|\alpha=1}\leq v^{s}_{m}\leq v^{s}\leq(a-c)$.

1. Consider $v_{NP}\leq v_{IP}\leq v^{s}_{m|\alpha=1}$. We get:

$$\begin{aligned}\pi_N^*(\alpha<1)-\pi_N^*(\alpha=1)&=\frac{[a-c-v_{NP}+m(v_{IP}-v_{NP})]^2}{4b(m+1)}-\frac{(n-m)[a-c-v_{NP}+m(v_{IP}-v_{NP})]^2}{b(n+1)^2}\\&=\frac{[a-c-v_{NP}+m(v_{IP}-v_{NP})]^2(n-2m-1)^2}{4b(m+1)(n+1)^2}\geq 0\end{aligned}$$

2. Consider $v^{s}_{m|\alpha=1}\leq v_{IP}\leq v^{s}_{m}$. We observe that $\pi_N^*(\alpha<1)$ is increasing in v_{IP}. Also, for $v_{IP}=v^{s}_{m|\alpha=1}$, we have:

$$\begin{aligned}\pi_N^*\left(\alpha<1|v_{IP}=v^{s}_{m|\alpha=1}\right)-\pi_N^*(\alpha=1)&=\frac{\left[a-c-v_{NP}+m\left(v^{s}_{m|\alpha=1}-v_{NP}\right)\right]^2}{4b(m+1)}-\frac{(n-m)[a-c-v_{NP}]^2}{b(n-m+1)^2}\\&=\frac{\left[a-c-v_{NP}+m\left(\dfrac{a-c-v_{NP}}{(n-m+1)}\right)\right]^2}{4b(m+1)}-\frac{(n-m)[a-c-v_{NP}]^2}{b(n-m+1)^2}\\&=\left[\frac{a-c-v_{NP}}{n-m+1}\right]^2\left[\frac{(n+1)^2}{4b(m+1)}-\frac{(n-m)}{b}\right]\\&=\left[\frac{a-c-v_{NP}}{n-m+1}\right]^2\left[\frac{(n-2m-1)^2}{4b(m+1)}\right]\geq 0\end{aligned}$$

Hence, for $v^{s}_{m|\alpha=1}\leq v_{IP}\leq v^{s}_{m}$, $\pi_N^*(\alpha<1)-\pi_N^*(\alpha=1)\geq 0$.

3. Consider $v^{s}_{m}\leq v_{IP}\leq v^{s}$. We observe that $\pi_N^*(\alpha<1)=\dfrac{(a-c-v_{IP})(v_{IP}-v_{NP})}{b(n-m)}$ is concave in v_{IP} with maximum occurring at $v_{IP}=v^{s}$. Therefore, $\pi_N^*(\alpha<1)$ is increasing in v_{IP} for $v_{IP}\leq v^{s}$. Also, for $v_{IP}=v^{s}_{m}$, we have:

$$\pi_N^*(\alpha<1|v_{IP}=v^{s}_{m})-\pi_N^*(\alpha=1)=\frac{(a-c-v^{s}_{m})(v^{s}_{m}-v_{NP})}{b}-\frac{(n-m)(a-c-v_{NP})^2}{b(n-m+1)^2}$$

$$= \frac{\left(a-c-v_{NP}-\dfrac{a-c-v_{NP}}{m+2}\right)\left(\dfrac{a-c-v_{NP}}{m+2}\right)}{b} - \frac{(n-m)(a-c-v_{NP})^2}{b(n-m+1)^2}$$
$$= \left[\frac{(a-c-v_{NP})^2}{b}\right]\left[\frac{(m+1)}{(m+2)^2} - \frac{(n-m)}{(n-m+1)^2}\right]$$
$$= \left[\frac{(a-c-v_{NP})^2}{b}\right](n-2m-1)\left[(n-m)(m+1)-1\right] \geq 0$$

Hence, for $v^s_m \leq v_{IP} \leq v^s_m$, $\pi^*_N(\alpha<1) - \pi^*_N(\alpha=1) \geq 0$.

4. Consider $v^s \leq v_{IP} \leq (a-c)$. We have:

$$\pi^*_N(\alpha<1) - \pi^*_N(\alpha=1) = \frac{1}{b}\left(\frac{a-c-v_{NP}}{2}\right)^2 - \frac{(n-m)(a-c-v_{NP})^2}{b(n-m+1)^2}$$
$$= \left[\frac{(a-c-v_{NP})^2}{b}\right]\left[\frac{1}{4} - \frac{(n-m)}{(n-m+1)^2}\right]$$
$$= \left[\frac{(a-c-v_{NP})^2}{b}\right]\left[\frac{(n-m-1)^2}{4(n-m+1)^2}\right] \geq 0$$

Therefore, for $m \leq \dfrac{n-1}{2}$, $\pi^*_N(\alpha<1) \geq \pi^*_N(\alpha=1)$.

Case 2. $m > \dfrac{n-1}{2}$

From (23) and (33), we note that:

$$v^s - v^s_{m|\alpha=1} = \frac{a-c-v_{NP}}{2} - \frac{a-c-v_{NP}}{n-m+1} = \frac{(a-c-v_{NP})(n-m-1)}{2(n-m+1)} \leq 0$$

Also from Case 1 above, $v^s_m \leq v^s_{m|\alpha=1}$, and hence $v^s_m \leq v^s_{m|\alpha=1} \leq v^s \leq (a-c)$.

1. Consider $v_{NP} \leq v_{IP} \leq v^s_m$. We get:

$$\pi^*_N(\alpha<1) - \pi^*_N(\alpha=1) = \frac{[a-c-v_{NP}+m(v_{IP}-v_{NP})]^2}{4b(m+1)} - \frac{(n-m)[a-c-v_{NP}+m(v_{IP}-v_{NP})]^2}{b(n+1)^2}$$
$$= \frac{[a-c-v_{NP}+m(v_{IP}-v_{NP})]^2(n-2m-1)^2}{4b(m+1)(n+1)^2} \geq 0$$

2. Consider $v^s_m \leq v_{IP} \leq v^s_{m|\alpha=1}$. As shown above, $\pi^*_N(\alpha<1)$ is increasing in v_{IP} for $v_{IP} \leq v^s$. Hence, from 1 above, it is straightforward to show that $\pi^*_N(\alpha<1) - \pi^*_N(\alpha=1) \geq 0$.

3. Consider $v^s_{m|\alpha=1} \leq v_{IP} \leq v^s$. We observe that $\pi^*_N(\alpha<1) = \dfrac{(a-c-v_{IP})(v_{IP}-v_{NP})}{b(n-m)}$ is concave in v_{IP} with maximum occurring at $v_{IP} = v^s$. Therefore, $\pi^*_N(\alpha<1)$ is increasing in v_{IP} for $v_{IP} \leq v^s$. Also, for $v_{IP} = v^s_{m|\alpha=1}$, we have:

$$\pi^*_N\left(\alpha<1|v_{IP}=v^s_{m|\alpha=1}\right) - \pi^*_N(\alpha=1)$$
$$= \frac{\left(a-c-v^s_{m|\alpha=1}\right)\left(v^s_{m|\alpha=1}-v_{NP}\right)}{b} - \frac{(n-m)(a-c-v_{NP})^2}{b(n-m+1)^2}$$
$$= \frac{\left(a-c-v_{NP}-\dfrac{a-c-v_{NP}}{n-m+1}\right)\left(\dfrac{a-c-v_{NP}}{n-m+1}\right)}{b} - \frac{(n-m)(a-c-v_{NP})^2}{b(n-m+1)^2}$$
$$= \left[\frac{(a-c-v_{NP})^2}{b}\right]\left[\frac{(n-m)}{(n-m+1)^2} - \frac{(n-m)}{(n-m+1)^2}\right] = 0$$

Hence, for $v^s_m \leq v_{IP} \leq v^s_m$, $\pi^*_N(\alpha<1) - \pi^*_N(\alpha=1) \geq 0$.

4. Consider $v^s \leq v_{IP} \leq (a-c)$. We have:

$$\begin{aligned}\pi_N^*(\alpha<1)-\pi_N^*(\alpha=1) &= \frac{1}{b}\left(\frac{a-c-v_{NP}}{2}\right)^2-\frac{(n-m)(a-c-v_{NP})^2}{b(n-m+1)^2}\\ &= \left[\frac{(a-c-v_{NP})^2}{b}\right]\left[\frac{1}{4}-\frac{(n-m)}{(n-m+1)^2}\right]\\ &= \left[\frac{(a-c-v_{NP})^2}{b}\right]\left[\frac{(n-m-1)^2}{4(n-m+1)^2}\right]\geq 0\end{aligned}$$

Therefore, for $m > \dfrac{n-1}{2}$, $\pi_N^*(\alpha<1) \geq \pi_N^*(\alpha=1)$. □

Proof of Proposition 8. From Corollary 2, the condition for internal stability of the network, i.e., $\pi_{NP}^*(m) > \pi_{IP}^*(m+1)$, is given as follows:

$$\alpha b\left(q_{NP}^*(m)\right)^2 > b\left(q_{IP}^*(m+1)\right)^2 \Rightarrow \alpha > \left[\frac{q_{IP}^*(m+1)}{q_{NP}^*(m)}\right]^2$$

Similarly, the condition for external stability of the network, i.e., $\pi_{IP}^*(m) > \pi_{NP}^*(m-1)$, is given as follows:

$$b\left(q_{IP}^*(m)\right)^2 > \alpha b\left(q_{NP}^*(m-1)\right)^2 \Rightarrow \alpha < \left[\frac{q_{IP}^*(m)}{q_{NP}^*(m-1)}\right]^2$$

Note that q_{NP}^* and q_{IP}^* are independent of α. □

Proof of Corollary 4. From Corollary 2, q_{NP}^* is increasing and q_{IP}^* is decreasing in m. Therefore, it is straightforward to show that $\alpha_l^s \leq \alpha_u^s$. □

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